

High Speed Communication Circuits and Systems
Lecture 12
Noise in Voltage Controlled Oscillators

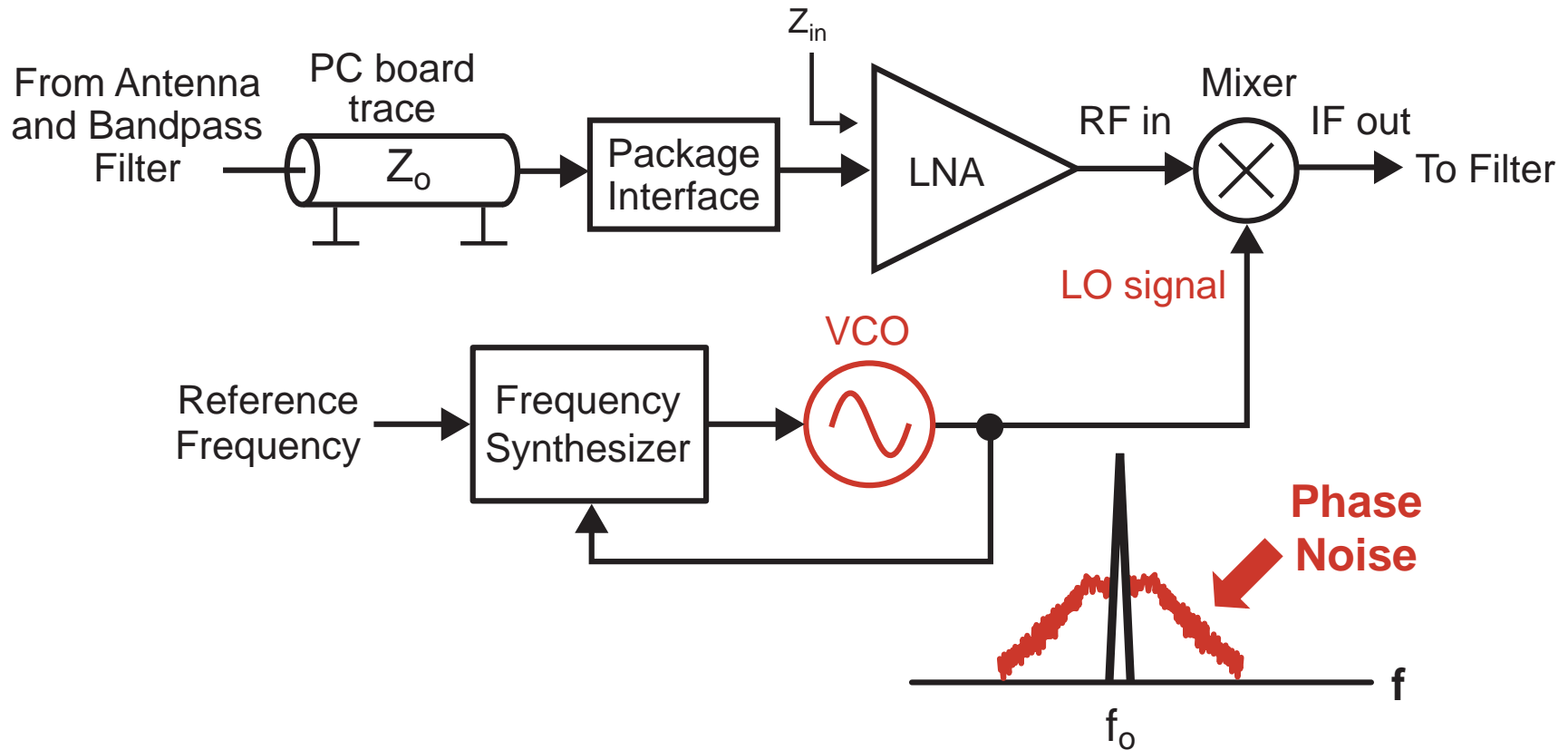
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March 12, 2004

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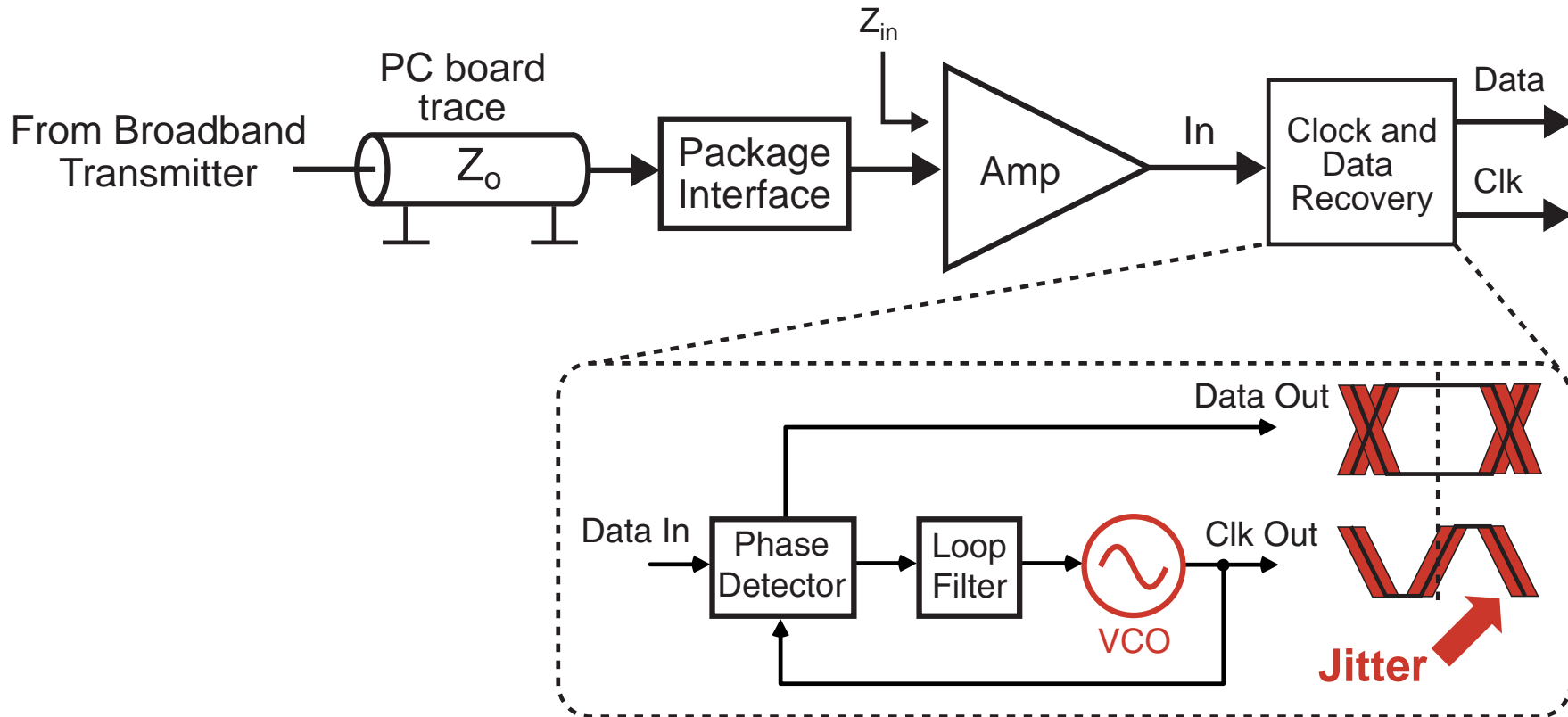
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VCO Noise in Wireless Systems



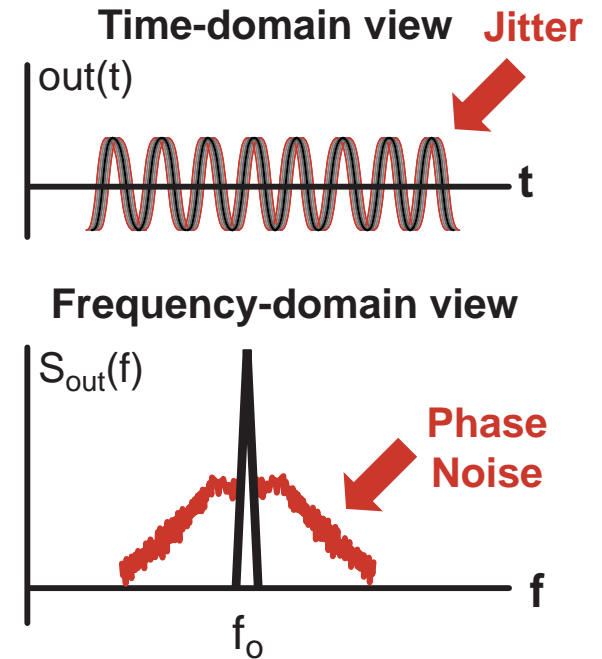
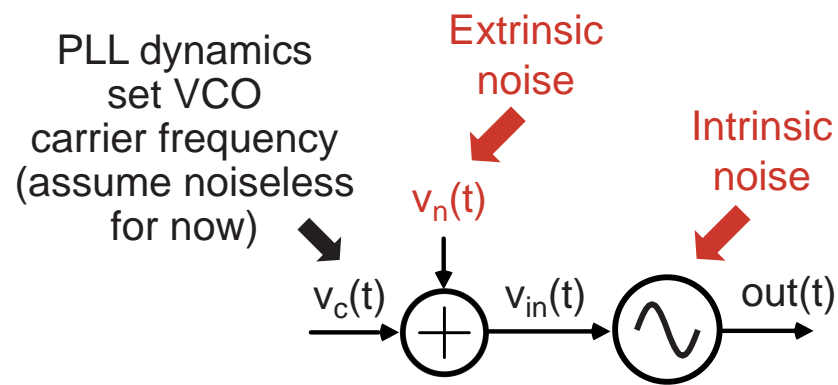
- **VCO noise has a negative impact on system performance**
 - Receiver – lower sensitivity, poorer blocking performance
 - Transmitter – increased spectral emissions (output spectrum must meet a mask requirement)
- **Noise is characterized in frequency domain**

VCO Noise in High Speed Data Links



- **VCO noise also has a negative impact on data links**
 - Receiver – increases bit error rate (BER)
 - Transmitter – increases jitter on data stream (transmitter must have jitter below a specified level)
- **Noise is characterized in the time domain**

Noise Sources Impacting VCO

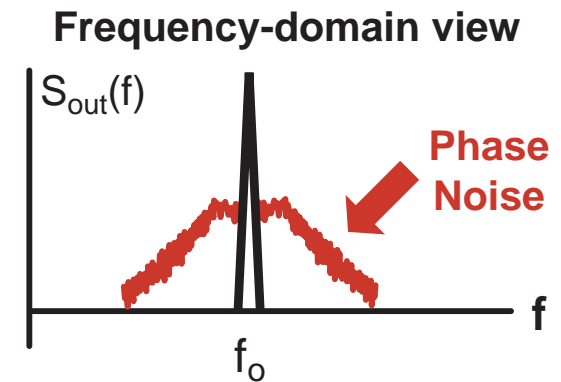
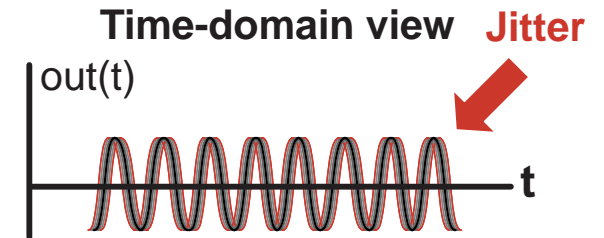
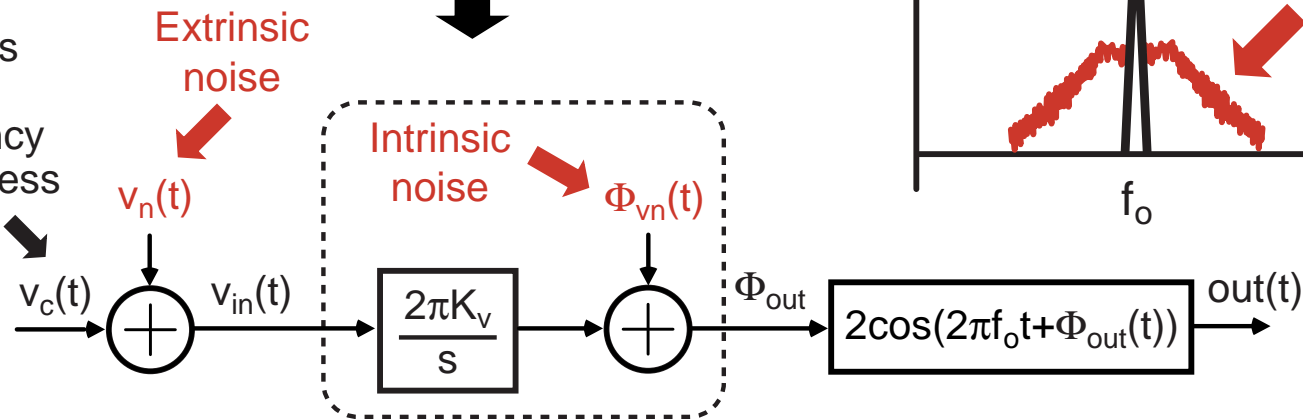


- **Extrinsic noise**
 - Noise from other circuits (including PLL)
- **Intrinsic noise**
 - Noise due to the VCO circuitry

VCO Model for Noise Analysis

Note: K_V units are Hz/V

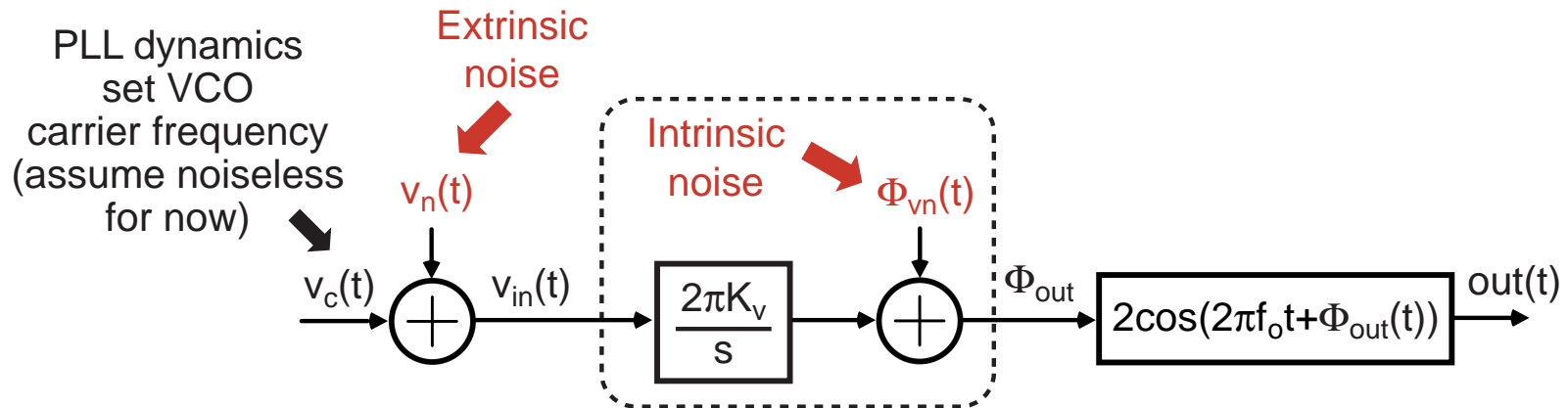
PLL dynamics set VCO carrier frequency (assume noiseless for now)



- We will focus on phase noise (and its associated jitter)
 - Model as phase signal in output sine waveform

$$out(t) = 2 \cos(2\pi f_0 t + \underline{\Phi_{out}(t)})$$

Simplified Relationship Between Φ_{out} and Output



$$out(t) = 2 \cos(2\pi f_o t + \Phi_{out}(t))$$

- Using a familiar trigonometric identity

$$out(t) = 2 \cos(2\pi f_o t) \cos(\Phi_{out}(t)) - 2 \sin(2\pi f_o t) \sin(\Phi_{out}(t))$$

- Given that the phase noise is small

$$\cos(\Phi_{out}(t)) \approx 1, \quad \sin(\Phi_{out}(t)) \approx \Phi_{out}(t)$$

$$\Rightarrow out(t) = 2 \cos(2\pi f_o t) - 2 \sin(2\pi f_o t) \Phi_{out}(t)$$

Calculation of Output Spectral Density

$$\underline{out(t) = 2 \cos(2\pi f_o t) - 2 \sin(2\pi f_o t) \Phi_{out}(t)}$$

- Calculate autocorrelation

$$R\{out(t)\} = R\{2 \cos(2\pi f_o t)\} + R\{2 \sin(2\pi f_o t)\} \cdot R\{\Phi_{out}(t)\}$$

- Take Fourier transform to get spectrum

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$

- Note that * symbol corresponds to convolution
- In general, phase spectral density can be placed into one of two categories
 - Phase noise – $\Phi_{out}(t)$ is non-periodic
 - Spurious noise - $\Phi_{out}(t)$ is periodic

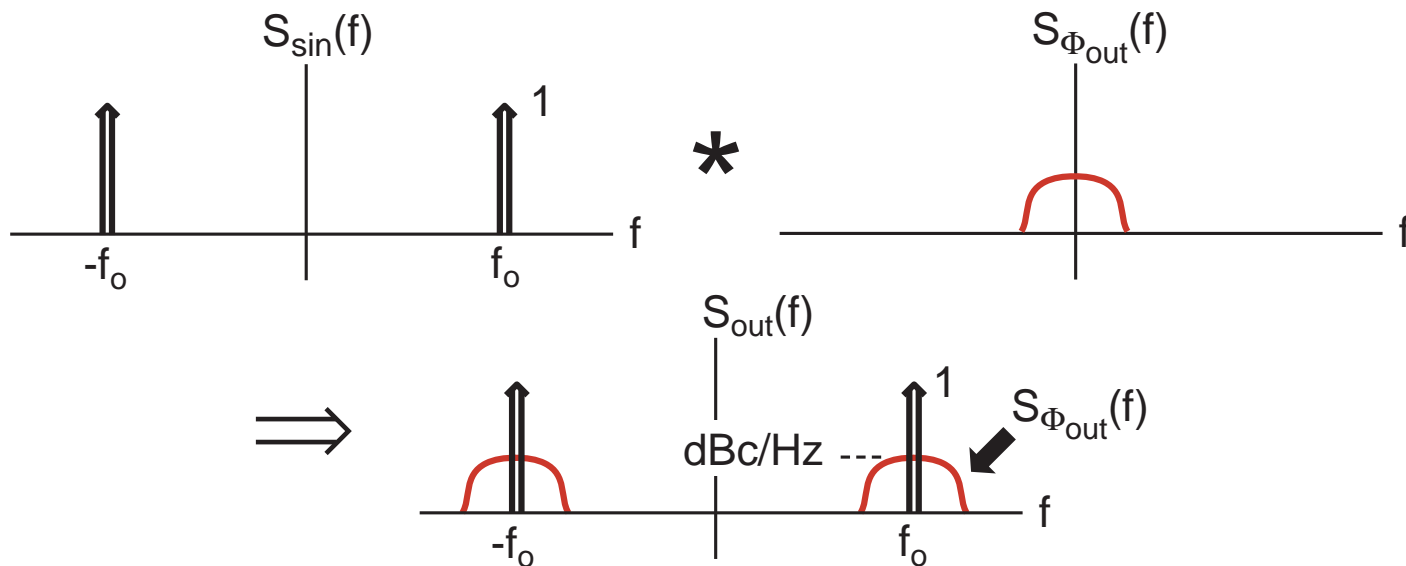
Output Spectrum with Phase Noise

- Suppose input noise to VCO ($v_n(t)$) is bandlimited, non-periodic noise with spectrum $S_{v_n}(f)$
 - In practice, derive phase spectrum as

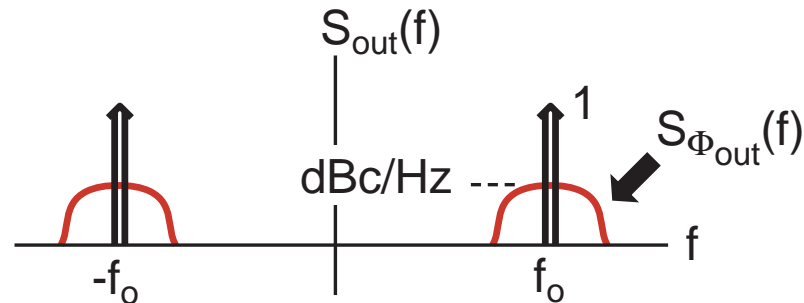
$$S_{\Phi_{out}}(f) = \left(\frac{K_v}{f}\right)^2 S_{v_n}(f)$$

- Resulting output spectrum

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Phase Noise in dBc/Hz



■ Definition of $L(f)$

$$L(f) = 10 \log \left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right)$$

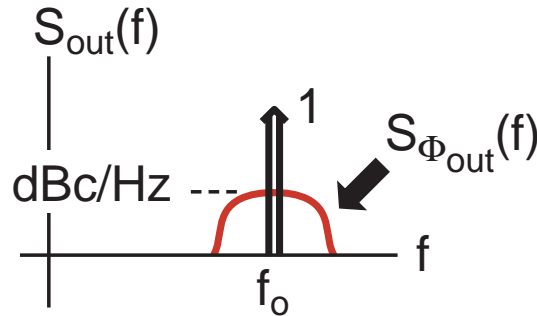
- Units are dBc/Hz

■ For this case

$$L(f) = 10 \log \left(\frac{2S_{\Phi_{out}}(f)}{2} \right) = 10 \log(S_{\Phi_{out}}(f))$$

- Valid when $\Phi_{out}(t)$ is small in deviation (i.e., when carrier is not modulated, as currently assumed)

Single-Sided Version



- **Definition of $L(f)$ remains the same**

$$L(f) = 10 \log \left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right)$$

- **Units are dBc/Hz**

- **For this case**

$$L(f) = 10 \log \left(\frac{S_{\Phi_{out}}(f)}{1} \right) = 10 \log(S_{\Phi_{out}}(f))$$

- **So, we can work with either one-sided or two-sided spectral densities since $L(f)$ is set by *ratio* of noise density to carrier power**

Output Spectrum with Spurious Noise

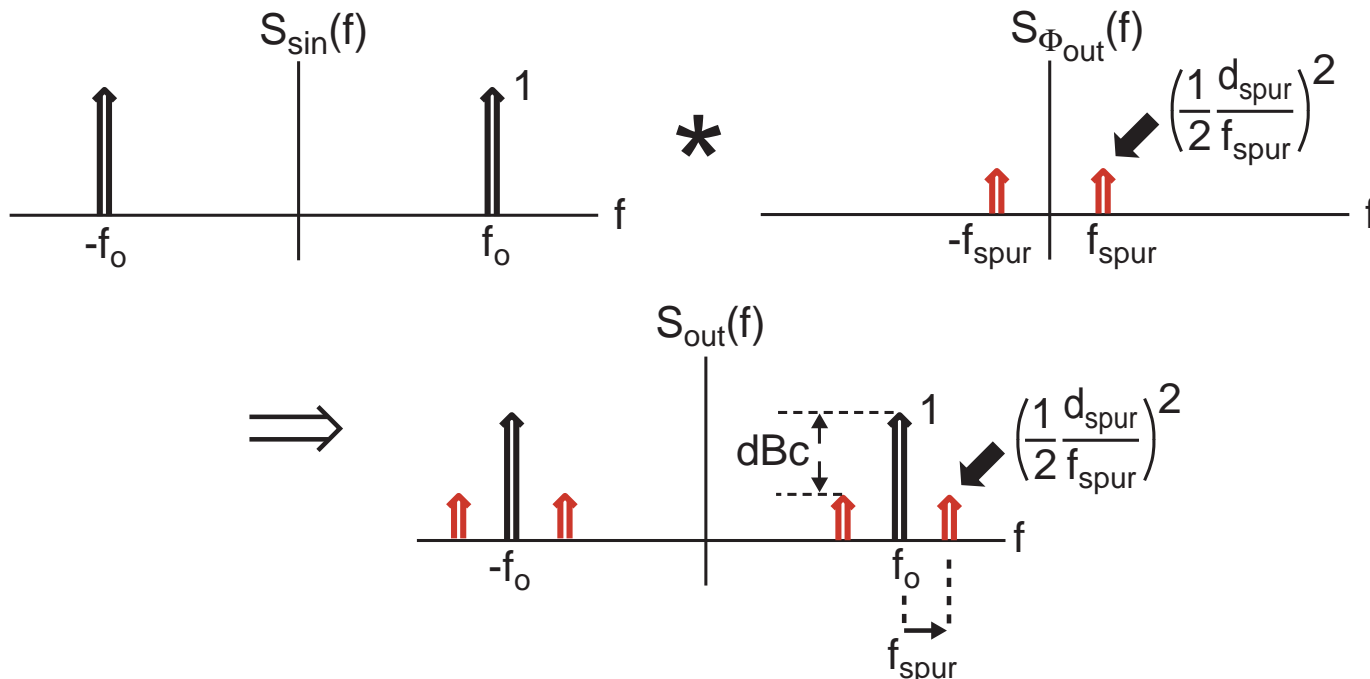
- Suppose input noise to VCO is

$$v_n(t) = \frac{d_{spur}}{K_v} \cos(2\pi f_{spur} t)$$

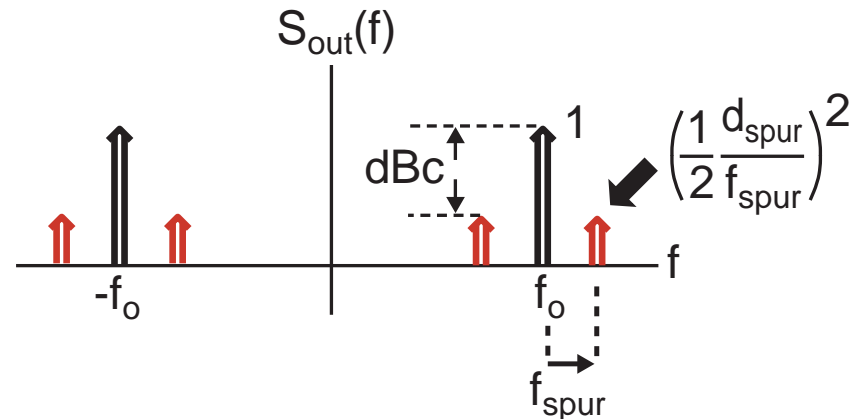
$$\Rightarrow \Phi_{out}(t) = 2\pi K_v \int v_n(t) dt = \frac{d_{spur}}{f_{spur}} \sin(2\pi f_{spur} t)$$

- Resulting output spectrum

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Spurious Noise in dBc



■ Definition of dBc

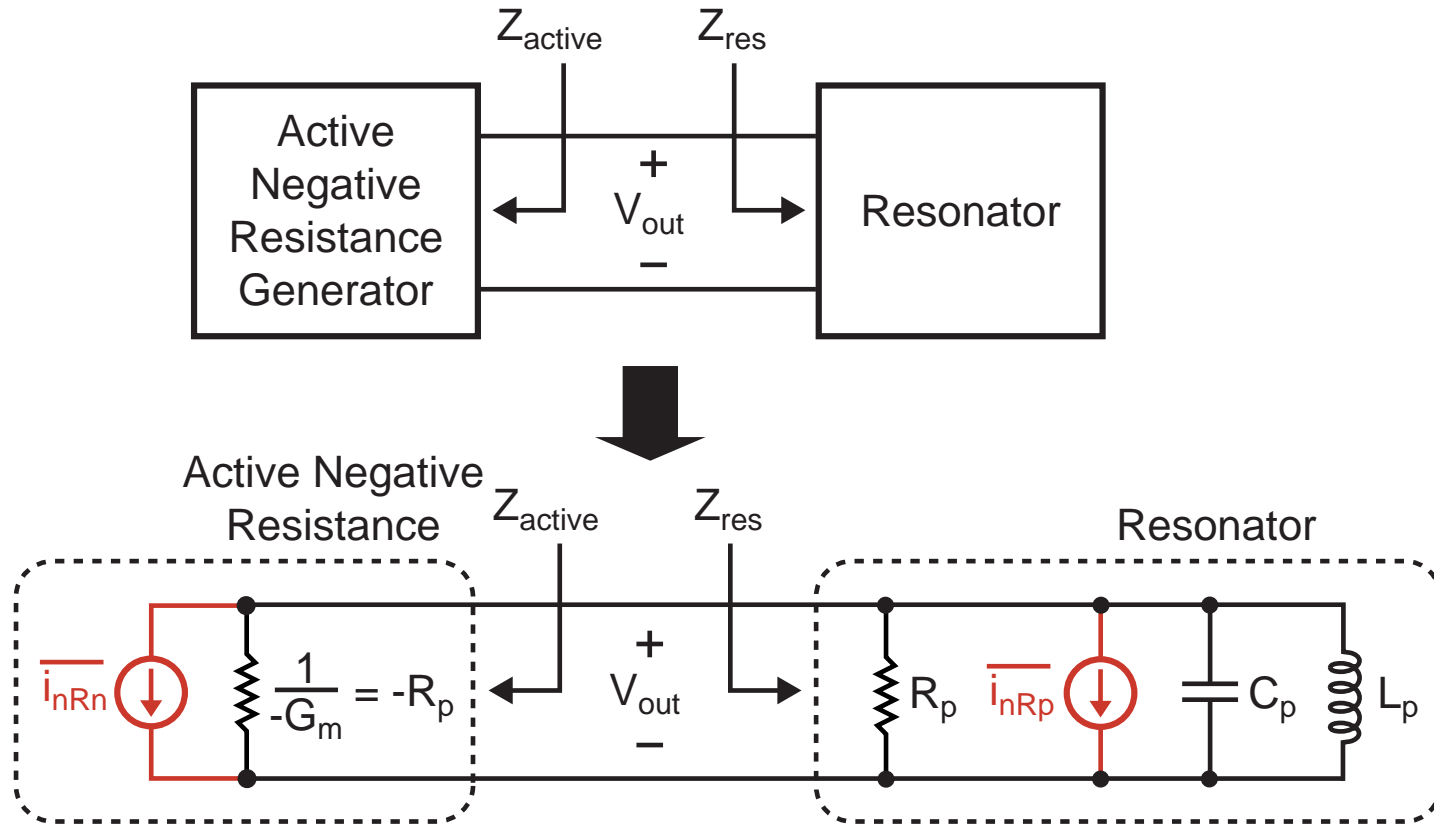
$$10 \log \left(\frac{\text{Power of tone}}{\text{Power of carrier}} \right)$$

- We are assuming double sided spectra, so integrate over positive and negative frequencies to get power
 - Either single or double-sided spectra can be used in practice

■ For this case

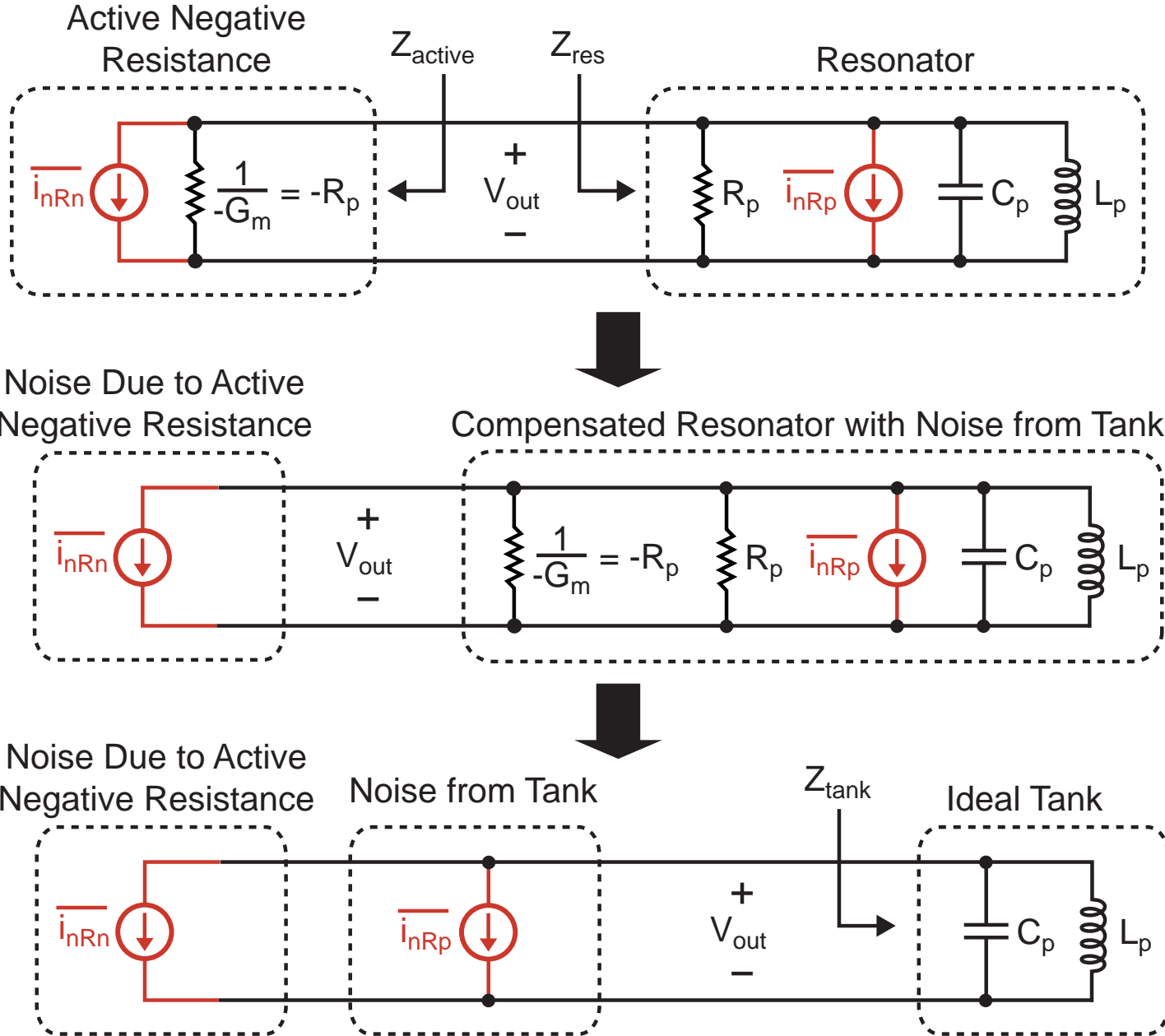
$$10 \log \left(\frac{2 \left(\frac{d_{spur}}{2 f_{spur}} \right)^2}{2} \right) = 20 \log \left(\frac{d_{spur}}{2 f_{spur}} \right) dBc$$

Calculation of Intrinsic Phase Noise in Oscillators

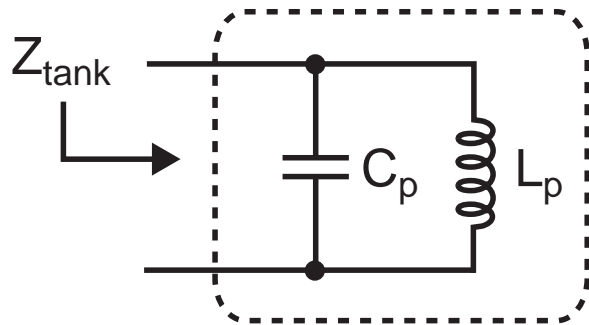


- Noise sources in oscillators are put in two categories
 - Noise due to tank loss
 - Noise due to active negative resistance
- We want to determine how these noise sources influence the phase noise of the oscillator

Equivalent Model for Noise Calculations



Calculate Impedance Across Ideal LC Tank Circuit



$$Z_{tank}(w) = \frac{1}{j\omega C_p} \parallel j\omega L_p = \frac{j\omega L_p}{1 - \omega^2 L_p C_p}$$

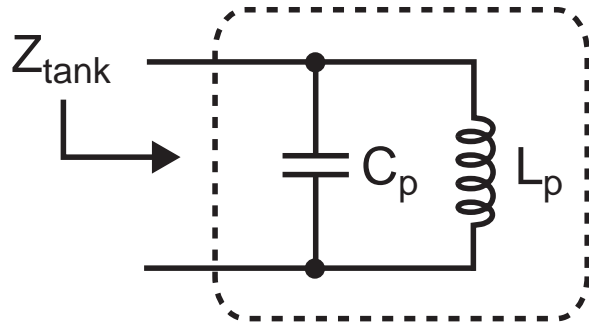
- Calculate input impedance about resonance

Consider $w = w_o + \Delta w$, where $w_o = \frac{1}{\sqrt{L_p C_p}}$

$$\begin{aligned} Z_{tank}(\Delta w) &= \frac{j(w_o + \Delta w)L_p}{1 - (w_o + \Delta w)^2 L_p C_p} \\ &= \frac{j(w_o + \Delta w)L_p}{\underbrace{1 - w_o^2 L_p C_p}_{= 0} - 2\Delta w(w_o L_p C_p) - \underbrace{\Delta w^2 L_p C_p}_{\text{negligible}}} \approx \frac{j(w_o + \Delta w)L_p}{-2\Delta w(w_o L_p C_p)} \end{aligned}$$

$$\Rightarrow Z_{tank}(\Delta w) \approx \frac{jw_o L_p}{-2\Delta w(w_o L_p C_p)} = \boxed{-\frac{j}{2w_o C_p} \left(\frac{w_o}{\Delta w} \right)}$$

A Convenient Parameterization of LC Tank Impedance



$$Z_{tank}(\Delta\omega) \approx -\frac{j}{2} \frac{1}{\omega_o C_p} \left(\frac{\omega_o}{\Delta\omega} \right)$$

- **Actual tank has loss that is modeled with R_p**
 - Define Q according to actual tank

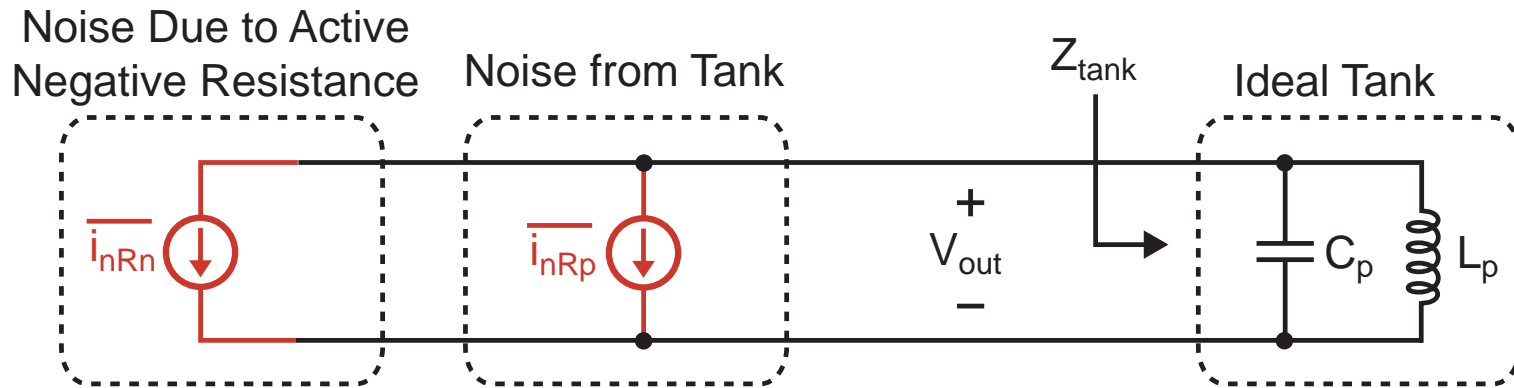
$$Q = R_p \omega_o C_p \Rightarrow \frac{1}{\omega_o C_p} = \frac{R_p}{Q}$$

- **Parameterize ideal tank impedance in terms of Q of actual tank**

$$Z_{tank}(\Delta\omega) \approx -\frac{j}{2} \frac{R_p}{Q} \left(\frac{\omega_o}{\Delta\omega} \right)$$

$$\Rightarrow |Z_{tank}(\Delta f)|^2 \approx \left(\frac{R_p f_o}{2Q \Delta f} \right)^2$$

Overall Noise Output Spectral Density

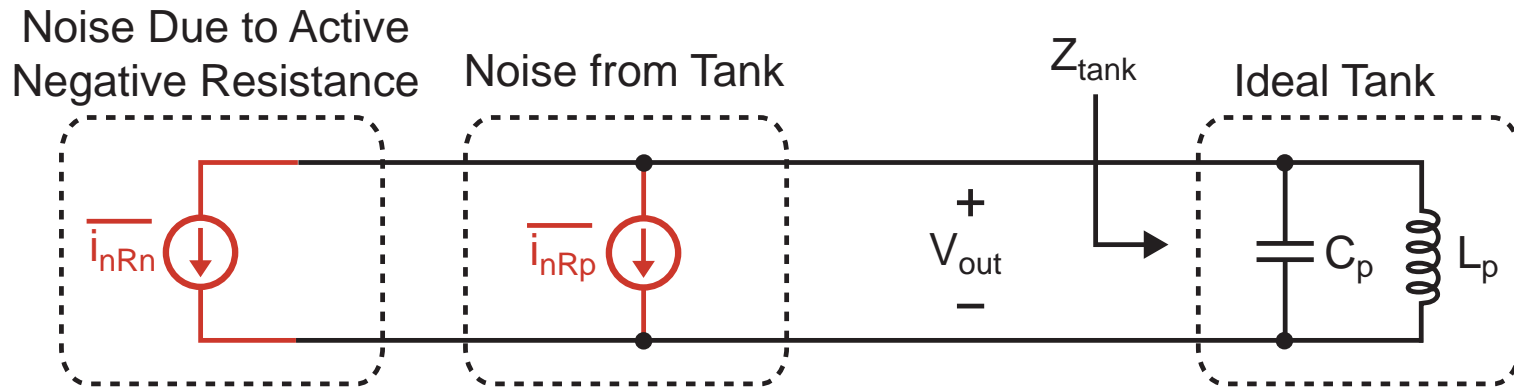


- Assume noise from active negative resistance element and tank are uncorrelated

$$\begin{aligned} \frac{\overline{v_{out}^2}}{\Delta f} &= \left(\frac{\overline{i_{nRp}^2}}{\Delta f} + \frac{\overline{i_{nRn}^2}}{\Delta f} \right) |Z_{tank}(\Delta f)|^2 \\ &= \frac{\overline{i_{nRp}^2}}{\Delta f} \left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} / \frac{\overline{i_{nRp}^2}}{\Delta f} \right) |Z_{tank}(\Delta f)|^2 \end{aligned}$$

- Note that the above expression represents total noise that impacts both amplitude and phase of oscillator output

Parameterize Noise Output Spectral Density



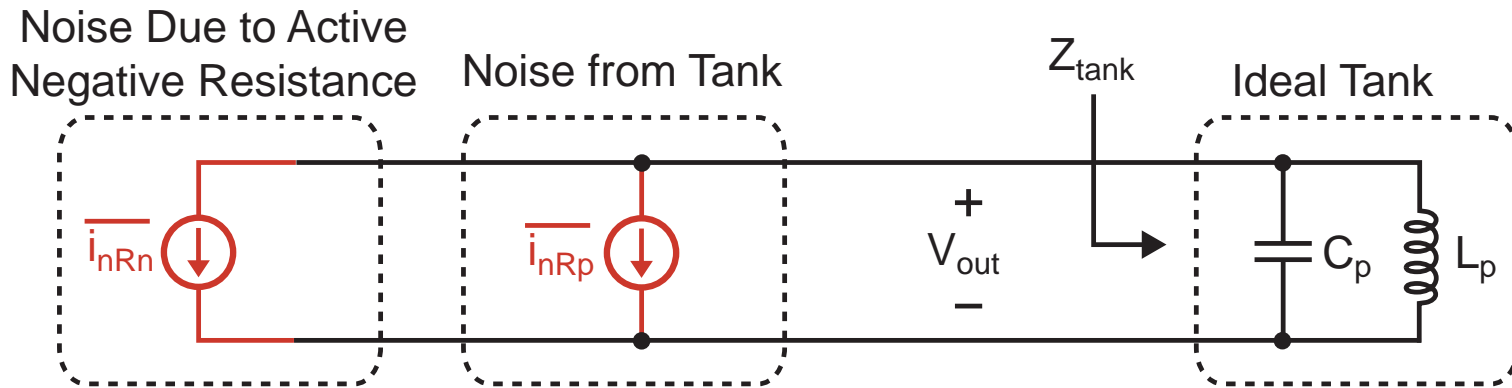
- From previous slide

$$\frac{\overline{v_{out}^2}}{\Delta f} = \frac{\overline{i_{nRp}^2}}{\Delta f} \underbrace{\left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} / \frac{\overline{i_{nRp}^2}}{\Delta f} \right)}_{F(\Delta f)} |Z_{tank}(\Delta f)|^2$$

- $F(\Delta f)$ is defined as

$$F(\Delta f) = \frac{\text{total noise in tank at frequency } \Delta f}{\text{noise in tank due to tank loss at frequency } \Delta f}$$

Fill in Expressions



- **Noise from tank is due to resistor R_p**

$$\frac{\overline{i_{nRp}^2}}{\Delta f} = 4kT \frac{1}{R_p} \quad (\text{single-sided spectrum})$$

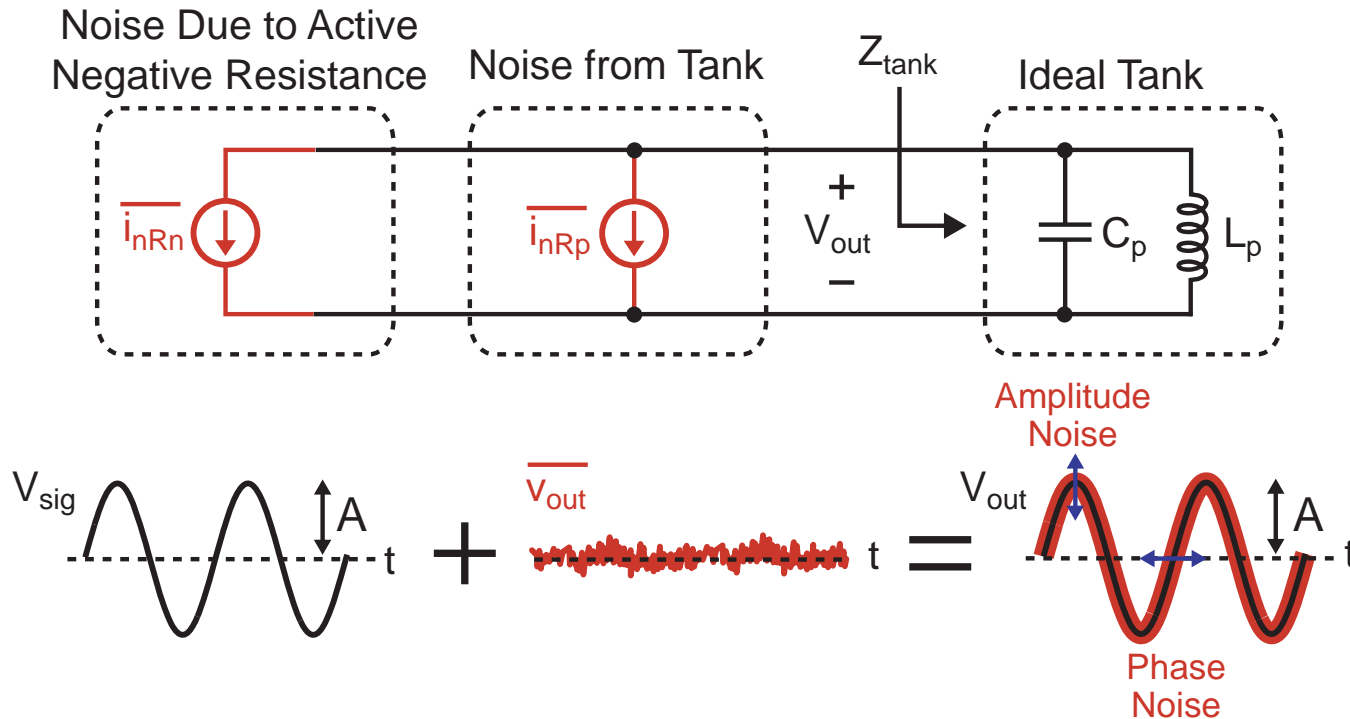
- **$Z_{tank}(\Delta f)$ found previously**

$$|Z_{tank}(\Delta f)|^2 \approx \left(\frac{R_p f_o}{2Q \Delta f} \right)^2$$

- **Output noise spectral density expression (single-sided)**

$$\frac{\overline{v_{out}^2}}{\Delta f} = 4kT \frac{1}{R_p} F(\Delta f) \left(\frac{R_p f_o}{2Q \Delta f} \right)^2 = 4kT F(\Delta f) R_p \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2$$

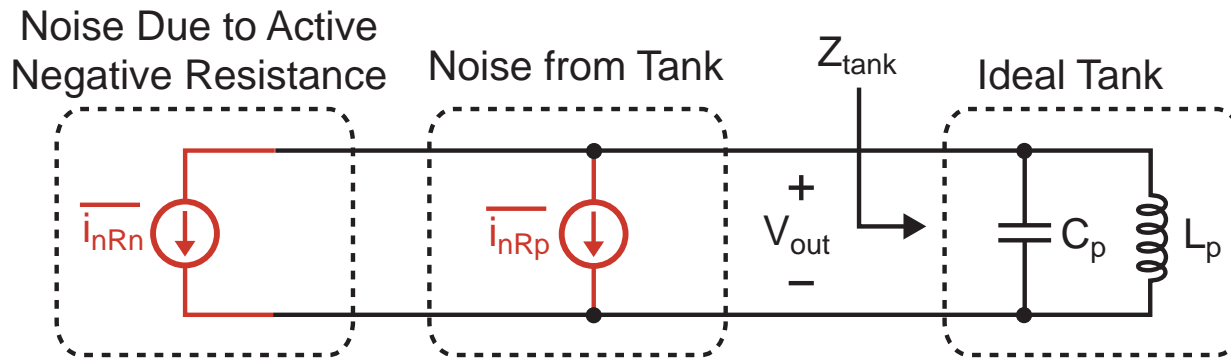
Separation into Amplitude and Phase Noise



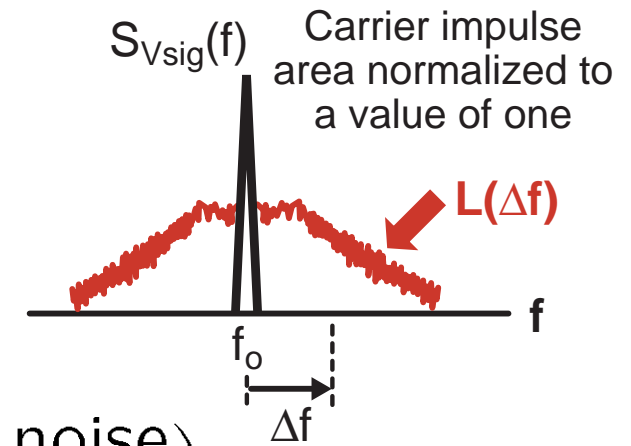
- Equipartition theorem (see Tom Lee, p 534) states that noise impact splits evenly between amplitude and phase for V_{sig} being a sine wave
 - Amplitude variations suppressed by feedback in oscillator

$$\Rightarrow \left. \frac{\overline{v_{out}^2}}{\Delta f} \right|_{\text{phase}} = 2kTF(\Delta f)R_p \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \quad (\text{single-sided})$$

Output Phase Noise Spectrum (Leeson's Formula)



Output Spectrum



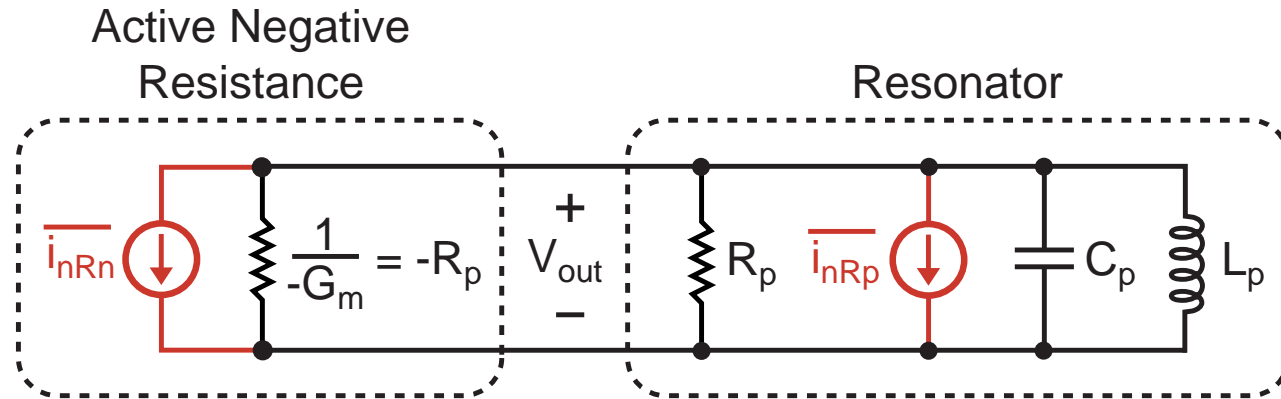
$$L(\Delta f) = 10 \log \left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right)$$

- All power calculations are referenced to the tank loss resistance, R_p

$$P_{sig} = \frac{V_{sig,rms}^2}{R_p} = \frac{(A/\sqrt{2})^2}{R_p}, \quad S_{noise}(\Delta f) = \frac{1}{R_p} \overline{v_{out}^2} \Delta f$$

$$L(\Delta f) = 10 \log \left(\frac{S_{noise}(\Delta f)}{P_{sig}} \right) = 10 \log \left(\frac{2kTF(\Delta f)}{P_{sig}} \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right)$$

Example: Active Noise Same as Tank Noise

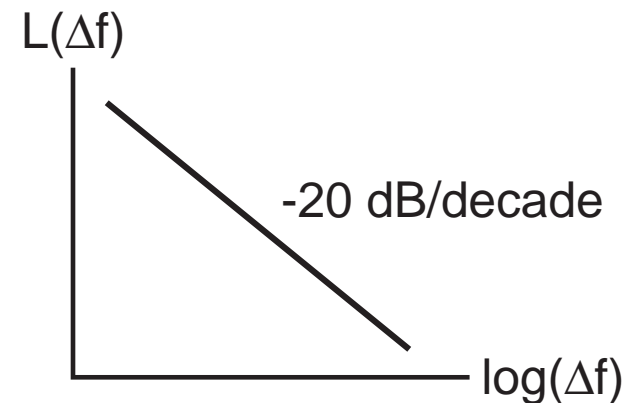


- Noise factor for oscillator in this case is

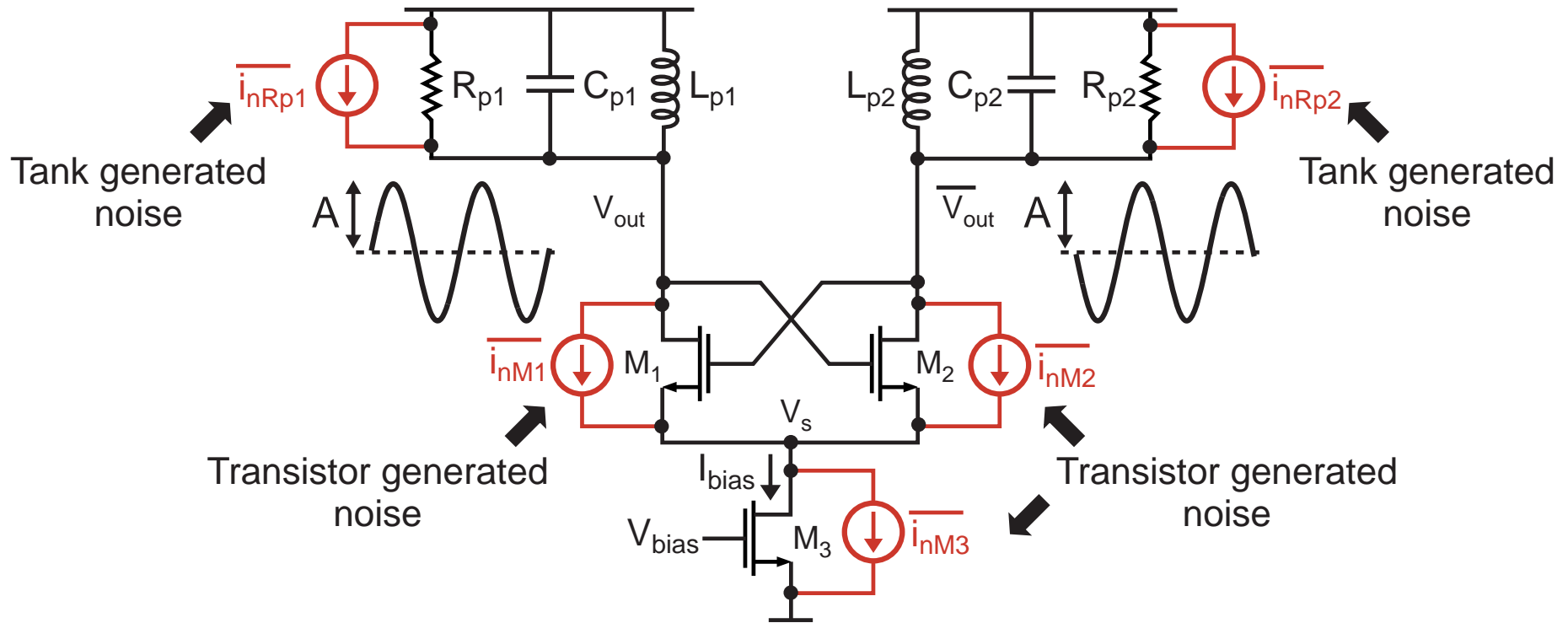
$$F(\Delta f) = 1 + \frac{\overline{i_{nRn}^2}}{\Delta f} / \frac{\overline{i_{nRp}^2}}{\Delta f} = 2$$

- Resulting phase noise

$$L(\Delta f) = 10 \log \left(\frac{4kT}{P_{sig}} \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right)$$

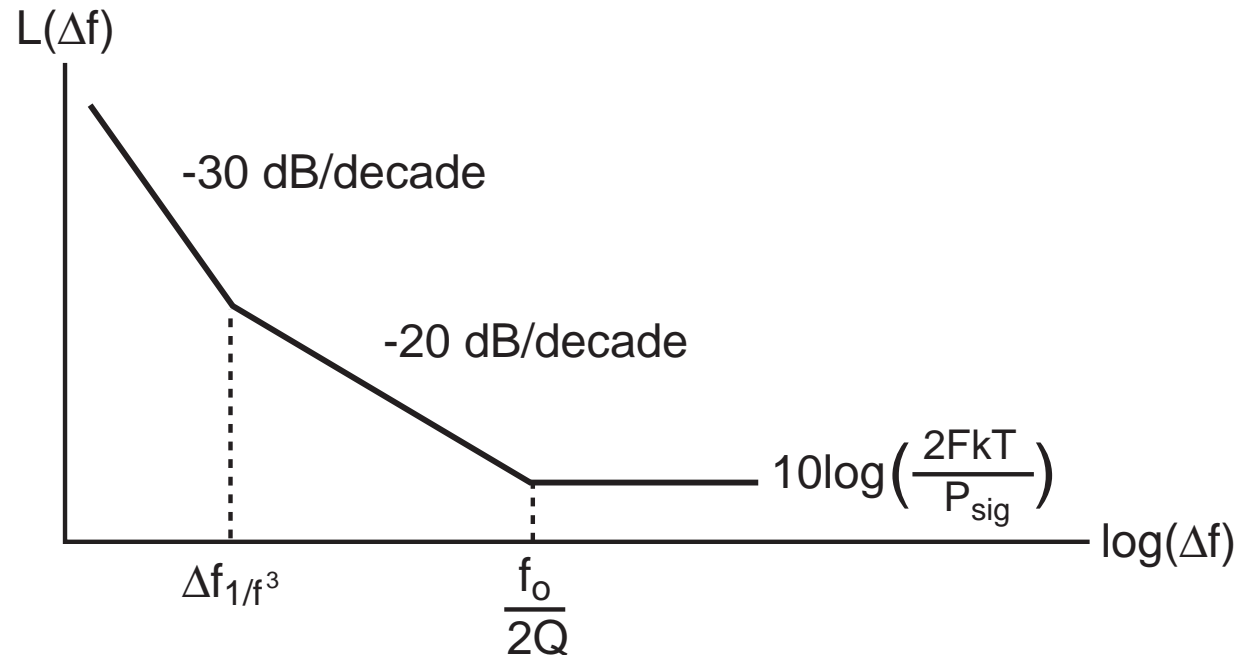


The Actual Situation is Much More Complicated



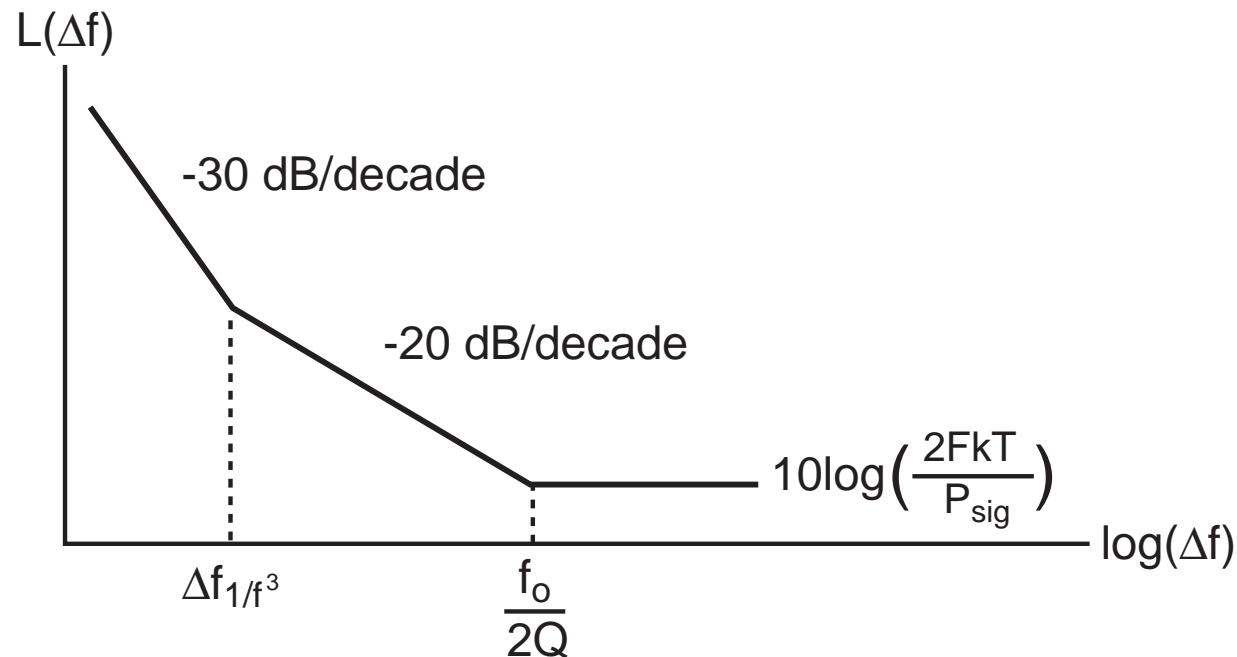
- Impact of tank generated noise easy to assess
- Impact of transistor generated noise is complicated
 - Noise from M_1 and M_2 is modulated on and off
 - Noise from M_3 is modulated before influencing V_{out}
 - Transistors have $1/f$ noise
- Also, transistors can degrade Q of tank

Phase Noise of A Practical Oscillator



- Phase noise drops at -20 dB/decade over a wide frequency range, but deviates from this at:
 - Low frequencies – slope increases (often -30 dB/decade)
 - High frequencies – slope flattens out (oscillator tank does not filter all noise sources)
- Frequency breakpoints and magnitude scaling are not readily predicted by the analysis approach taken so far

Phase Noise of A Practical Oscillator

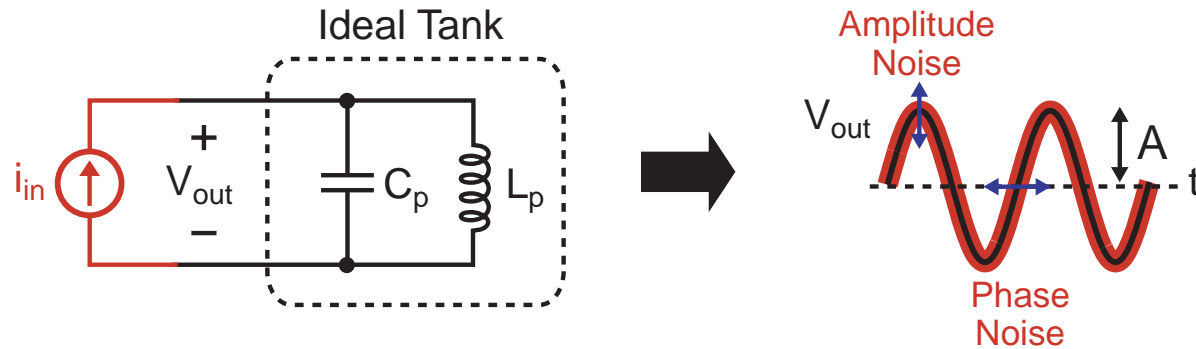


- Leeson proposed an ad hoc modification of the phase noise expression to capture the above noise profile

$$L(\Delta f) = 10 \log \left(\frac{2FkT}{P_{sig}} \left(1 + \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right) \left(1 + \frac{\Delta f_{1/f^3}}{|\Delta f|} \right) \right)$$

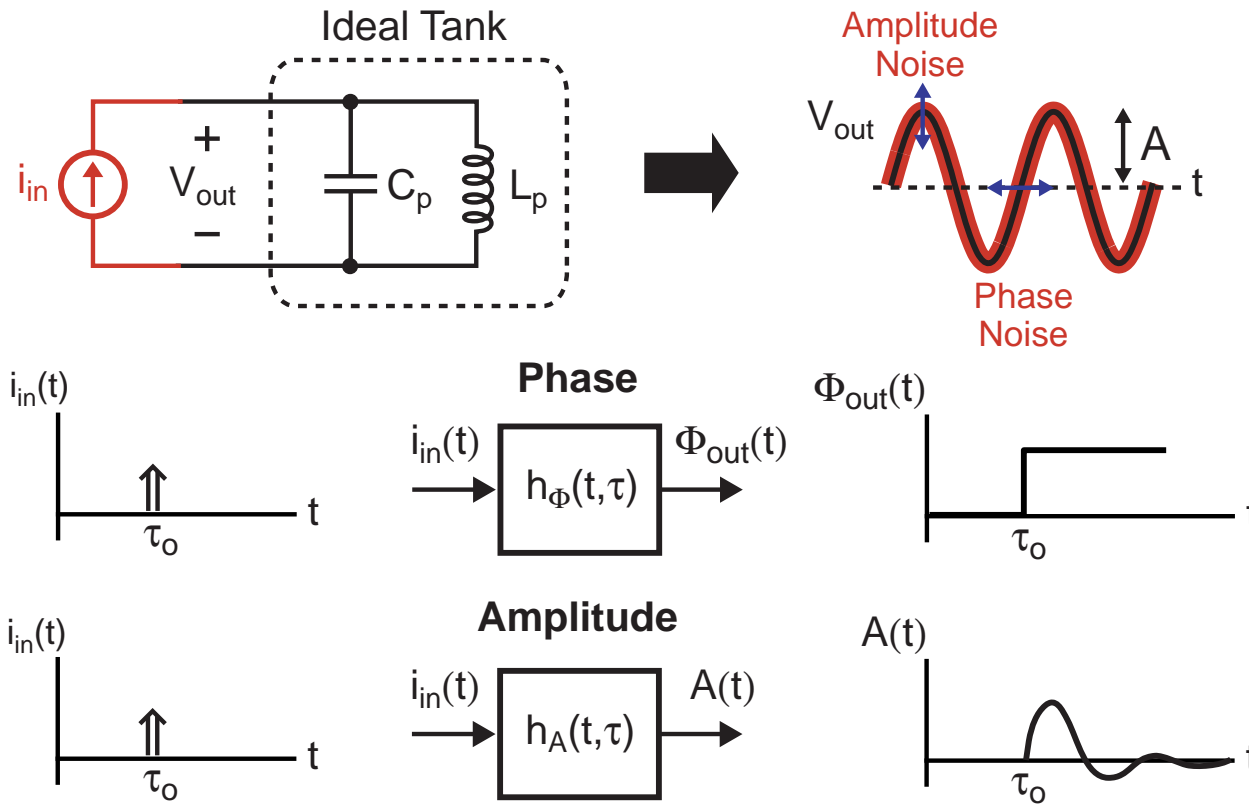
- Note: he assumed that $F(\Delta f)$ was constant over frequency

A More Sophisticated Analysis Method



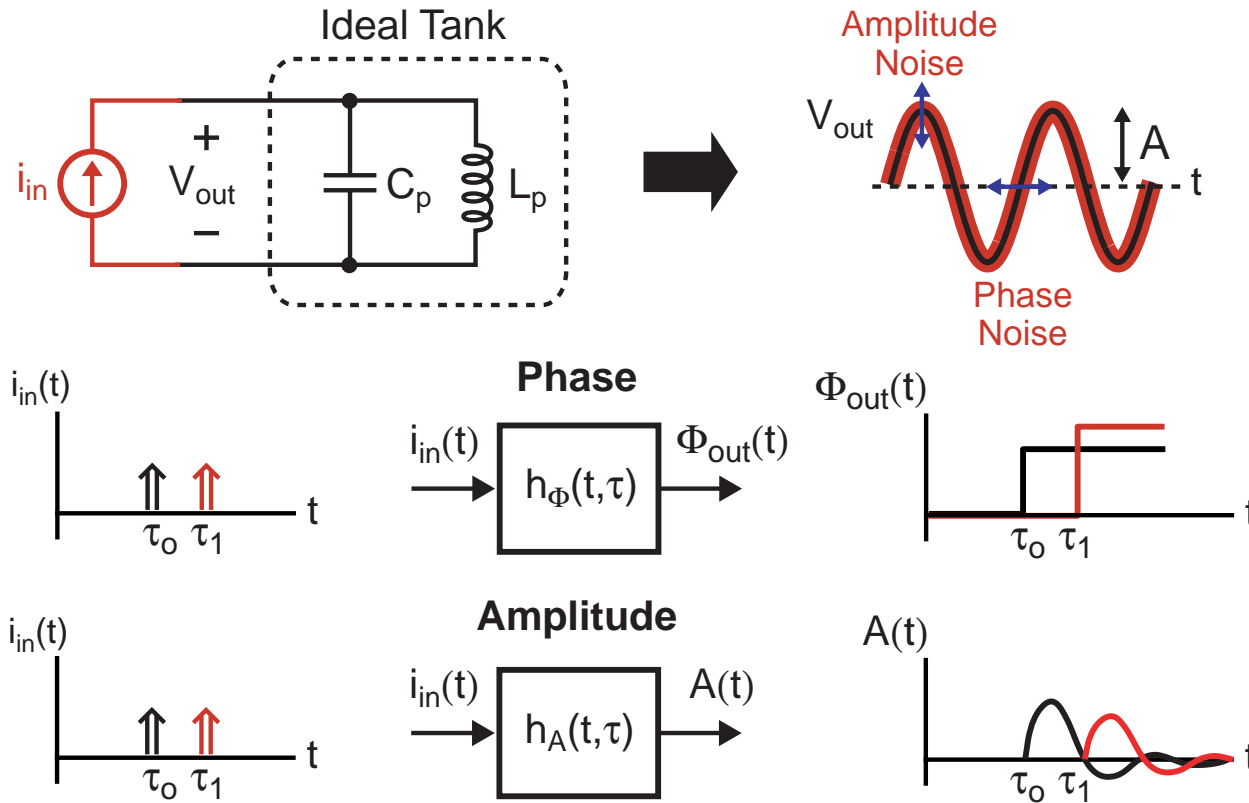
- Our concern is what happens when noise current produces a voltage across the tank
 - Such voltage deviations give rise to both amplitude and phase noise
 - Amplitude noise is suppressed through feedback (or by amplitude limiting in following buffer stages)
 - Our main concern is phase noise
- We argued that impact of noise divides equally between amplitude and phase for sine wave outputs
 - What happens when we have a non-sine wave output?

Modeling of Phase and Amplitude Perturbations



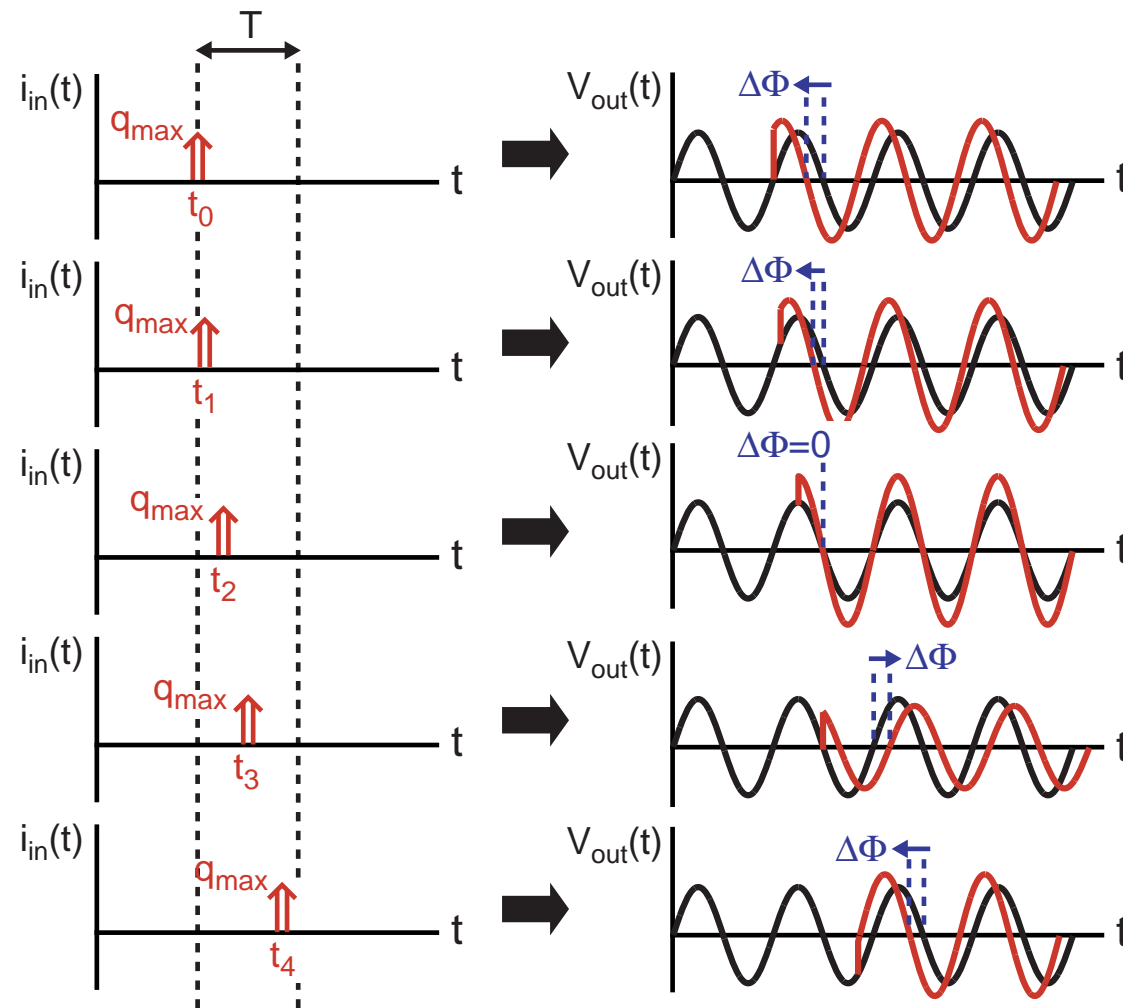
- Characterize impact of current noise on amplitude and phase through their associated impulse responses
 - Phase deviations are accumulated
 - Amplitude deviations are suppressed

Impact of Noise Current is Time-Varying



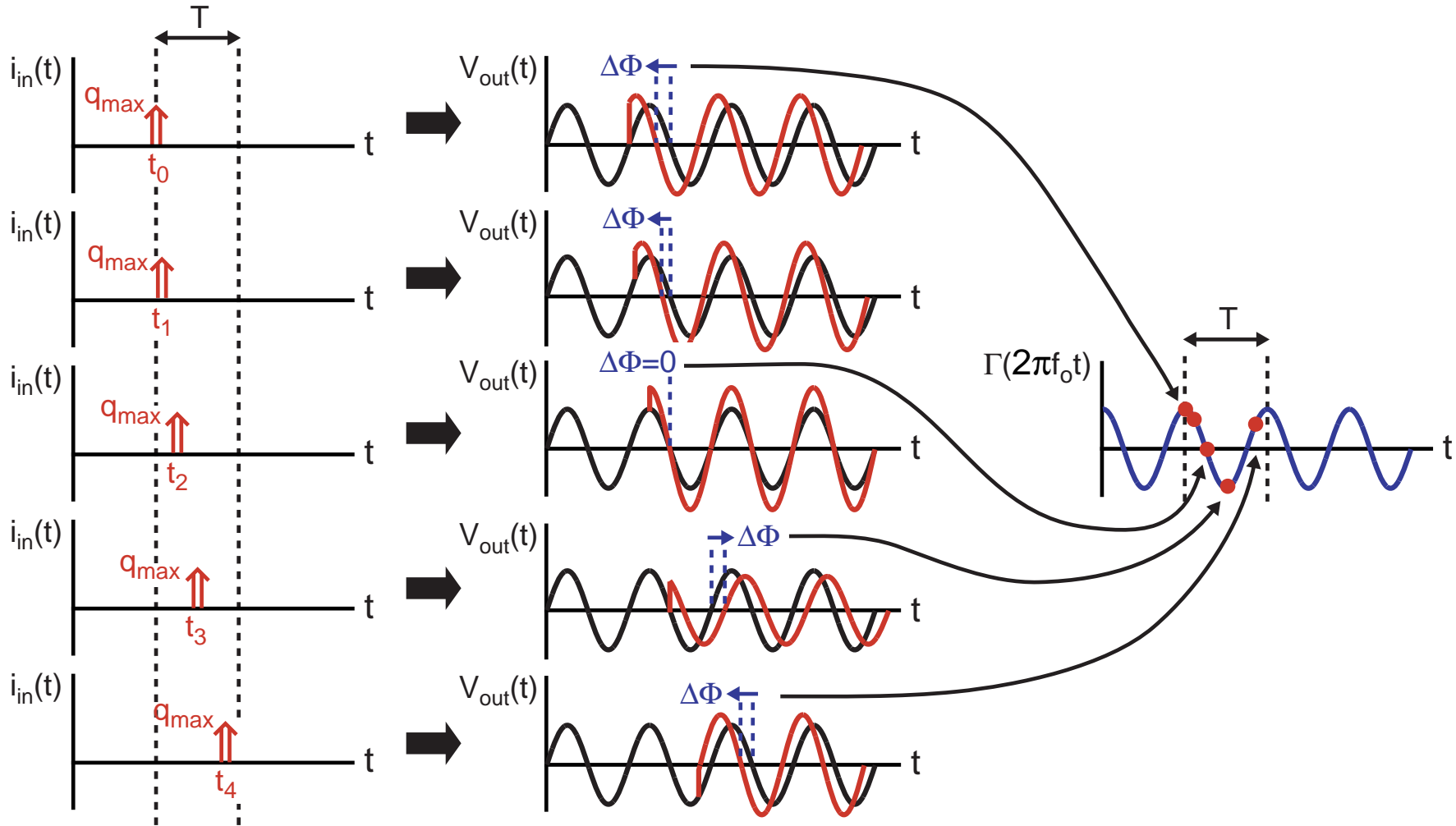
- If we vary the time at which the current impulse is injected, its impact on phase and amplitude changes
 - Need a time-varying model

Illustration of Time-Varying Impact of Noise on Phase



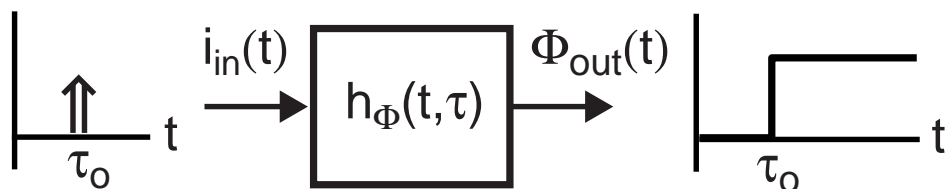
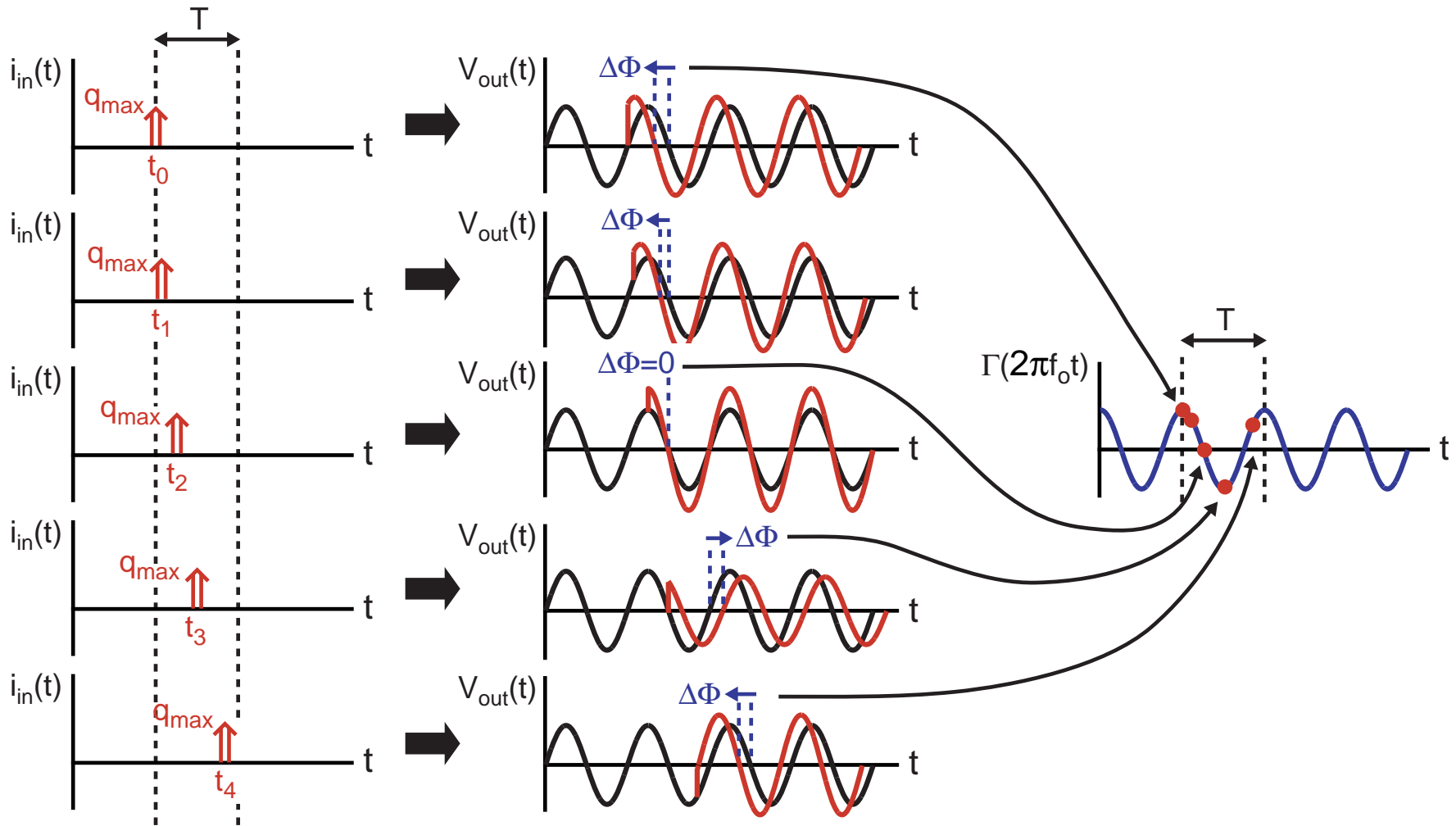
- High impact on phase when impulse occurs close to the zero crossing of the VCO output
- Low impact on phase when impulse occurs at peak of output

Define Impulse Sensitivity Function (ISF) – $\Gamma(2\pi f_o t)$



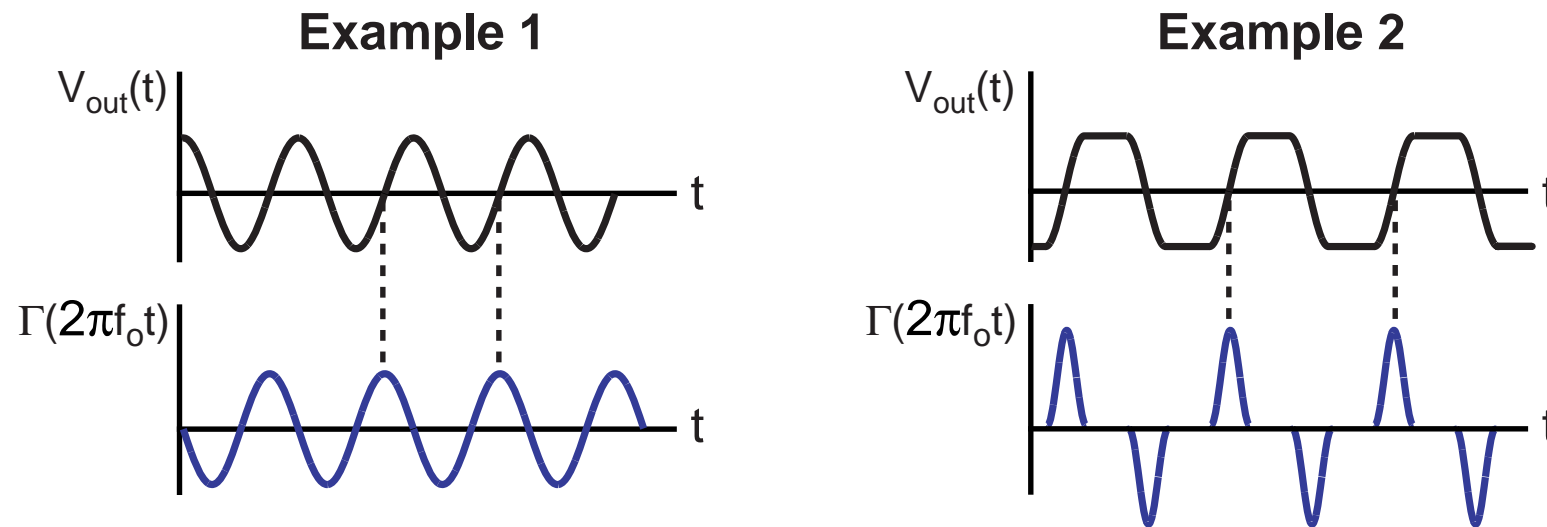
- **ISF constructed by calculating phase deviations as impulse position is varied**
 - Observe that it is periodic with same period as VCO output

Parameterize Phase Impulse Response in Terms of ISF



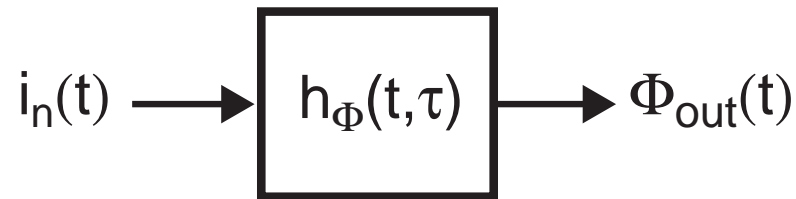
$$h_\Phi(t, \tau) = \frac{\Gamma(2\pi f_0 \tau)}{q_{max}} u(t - \tau)$$

Examples of ISF for Different VCO Output Waveforms



- **ISF (i.e., Γ) is approximately proportional to derivative of VCO output waveform**
 - Its magnitude indicates where VCO waveform is most sensitive to noise current into tank with respect to creating phase noise
- **ISF is periodic**
- **In practice, derive it from simulation of the VCO**

Phase Noise Analysis Using LTV Framework



- **Computation of phase deviation for an arbitrary noise current input**

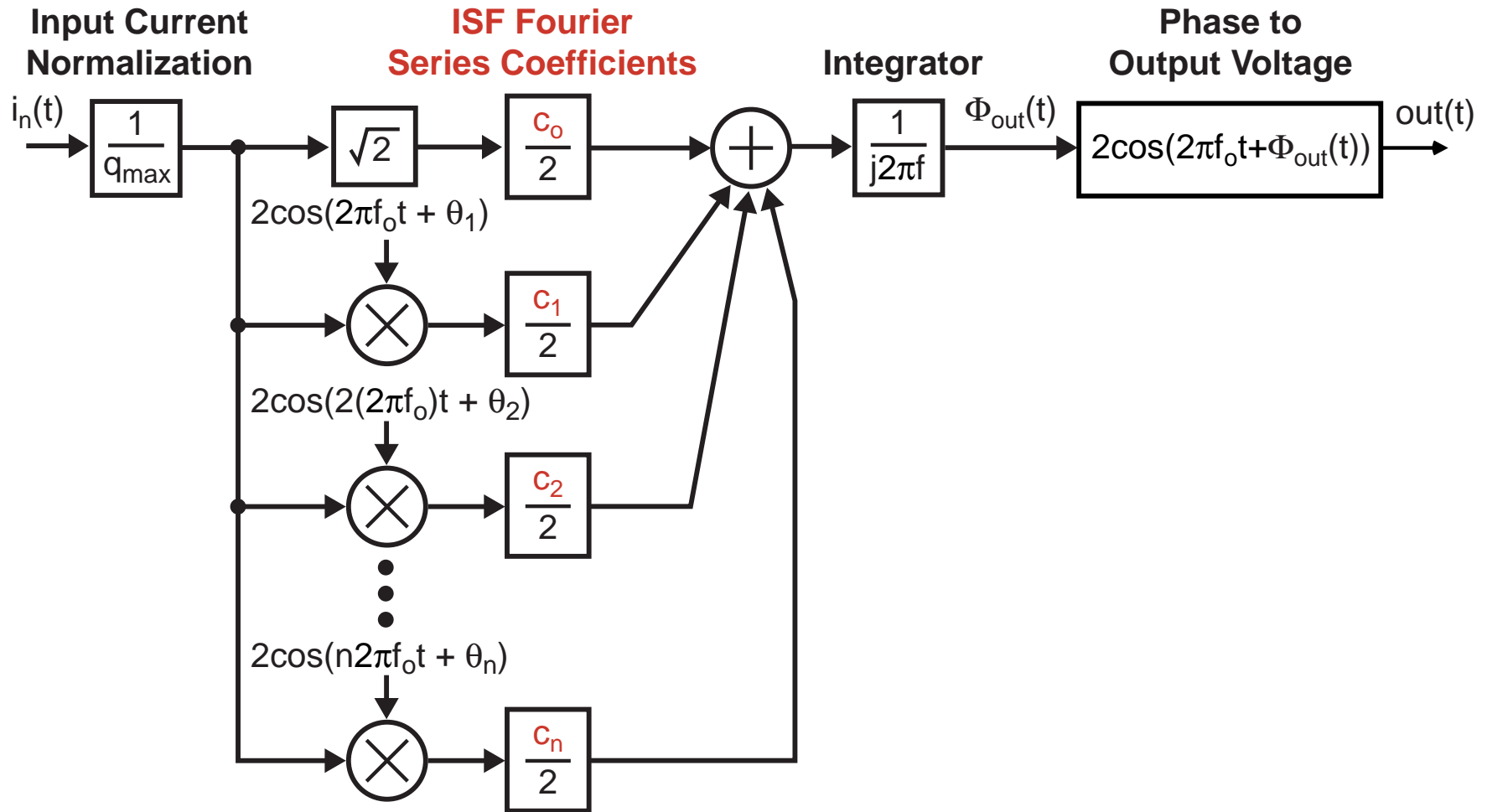
$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_{\Phi}(t, \tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^t \Gamma(2\pi f_o \tau) i_n(\tau) d\tau$$

- **Analysis simplified if we describe ISF in terms of its Fourier series (note: c_o here is different than book)**

$$\Gamma(2\pi f_o \tau) = \frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n2\pi f_o \tau + \theta_n)$$

$$\Rightarrow \Phi_{out}(t) = \int_{-\infty}^t \left(\frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n2\pi f_o \tau + \theta_n) \right) \frac{i_n(\tau)}{q_{max}} d\tau$$

Block Diagram of LTV Phase Noise Expression

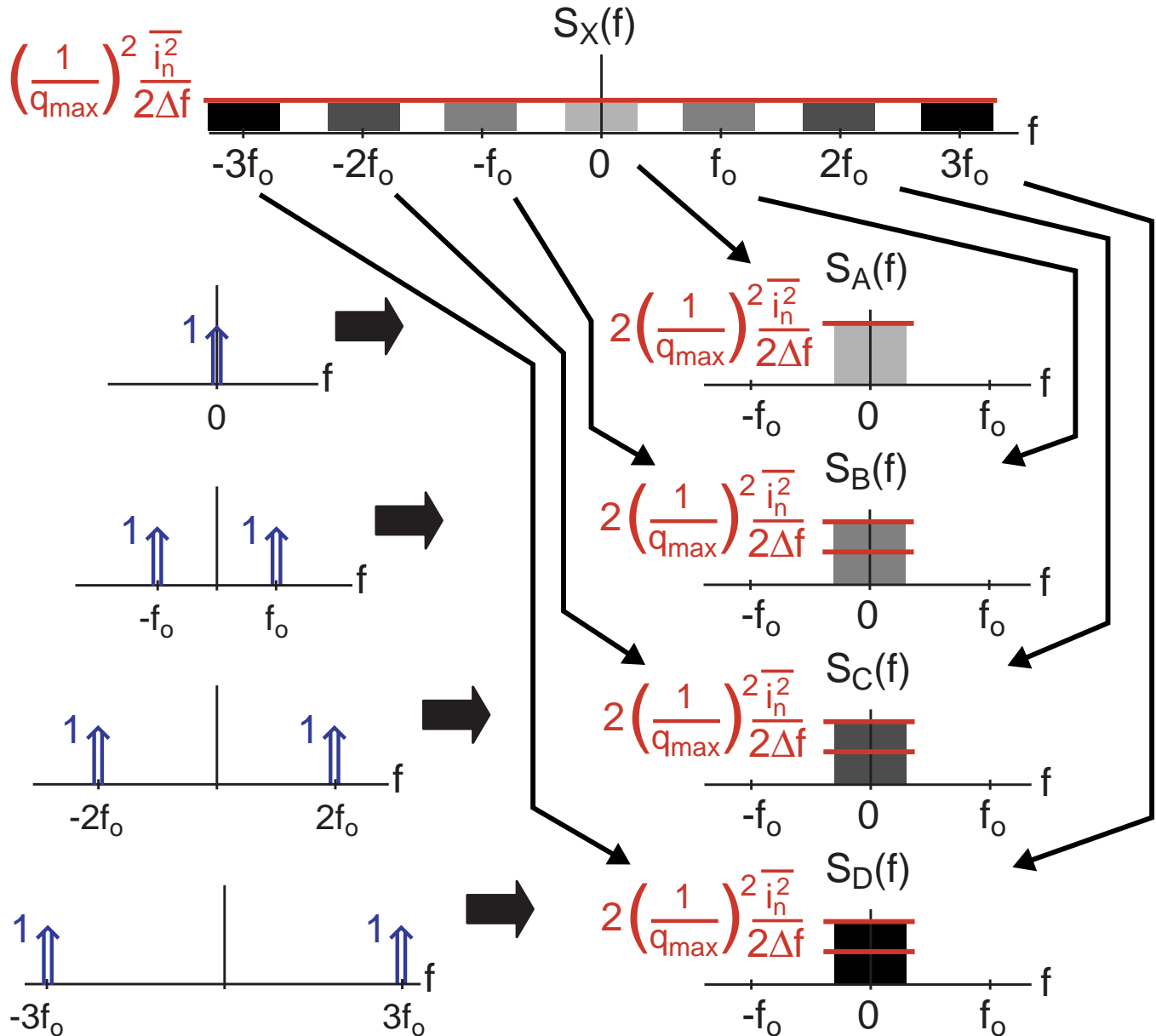
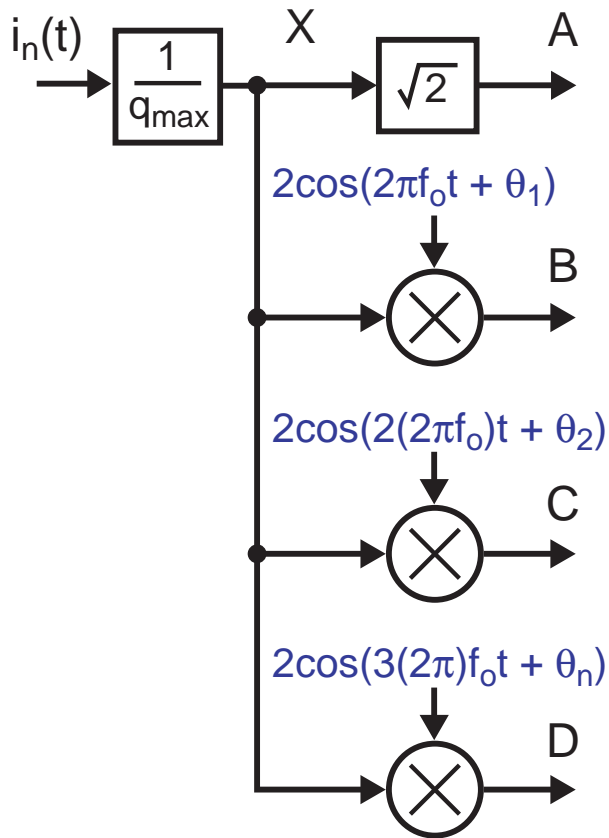


- Noise from current source is mixed down from different frequency bands and scaled according to ISF coefficients

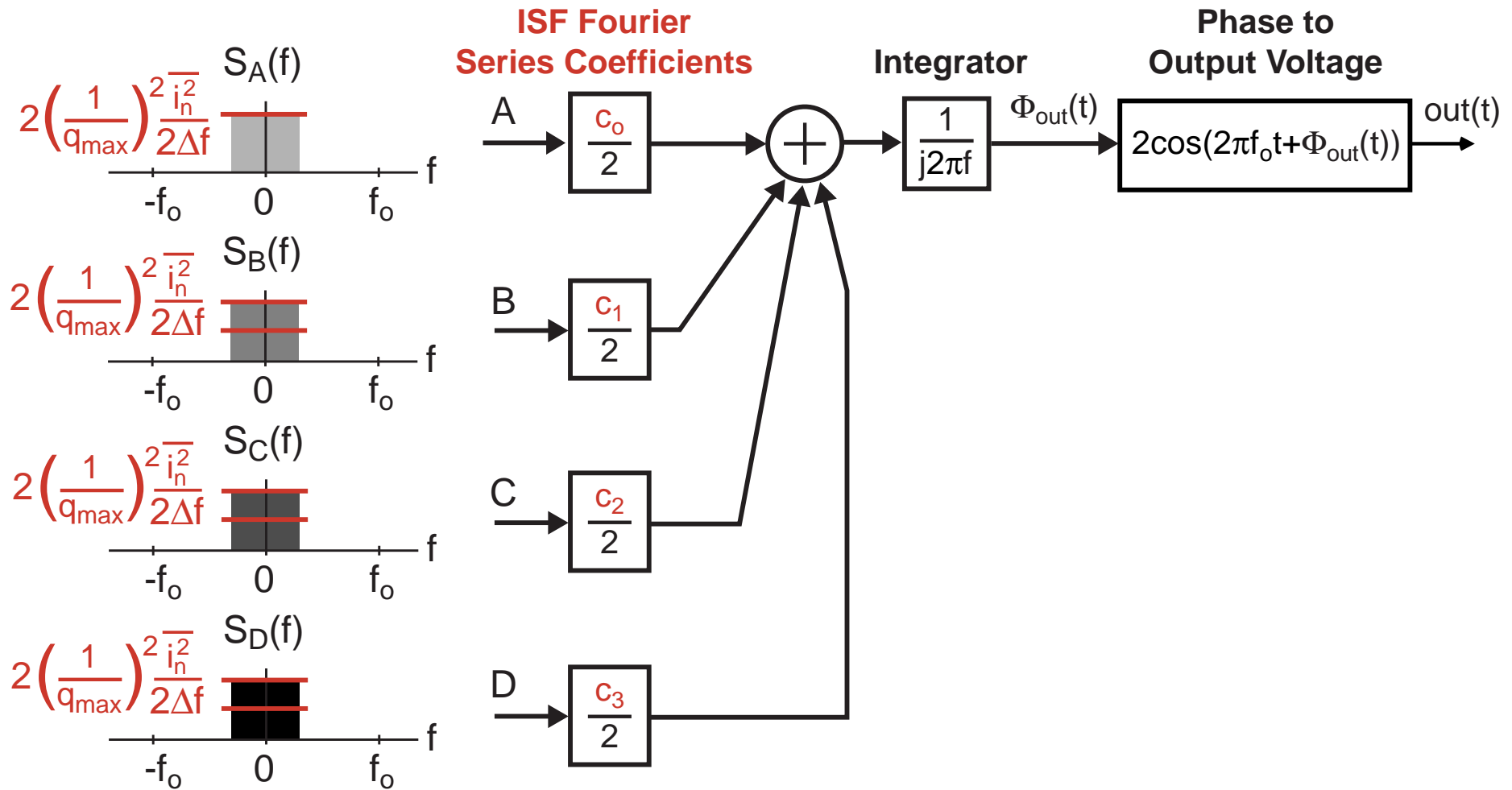
Phase Noise Calculation for White Noise Input (Part 1)

Note that $\frac{\overline{i_n^2}}{\Delta f}$ is the single-sided noise spectral density of $i_n(t)$

$$\left(\frac{1}{q_{\max}}\right)^2 \frac{\overline{i_n^2}}{2\Delta f}$$



Phase Noise Calculation for White Noise Input (Part 2)



$$S_{\Phi_{out}}(f) = \left| \frac{1}{j2\pi f} \right|^2 \left(\left(\frac{c_0}{2} \right)^2 S_A(f) + \left(\frac{c_1}{2} \right)^2 S_B(f) + \dots \right)$$

Spectral Density of Phase Signal

- From the previous slide

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_0}{2}\right)^2 S_A(f) + \left(\frac{c_1}{2}\right)^2 S_B(f) + \dots \right)$$

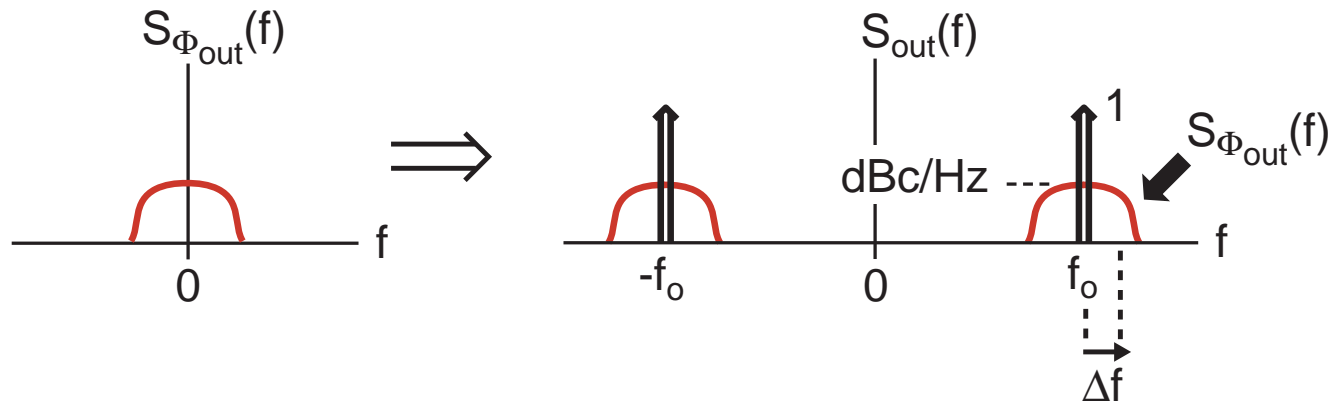
- Substitute in for $S_A(f)$, $S_B(f)$, etc.

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_0}{2}\right)^2 + \left(\frac{c_1}{2}\right)^2 + \dots \right) 2 \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{2\Delta f}$$

- Resulting expression

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

Output Phase Noise



- We now know**

$$S_{\Phi_{out}}(f) = \left| \frac{1}{2\pi f} \right|^2 \left(\sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left(\frac{1}{q_{max}} \right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

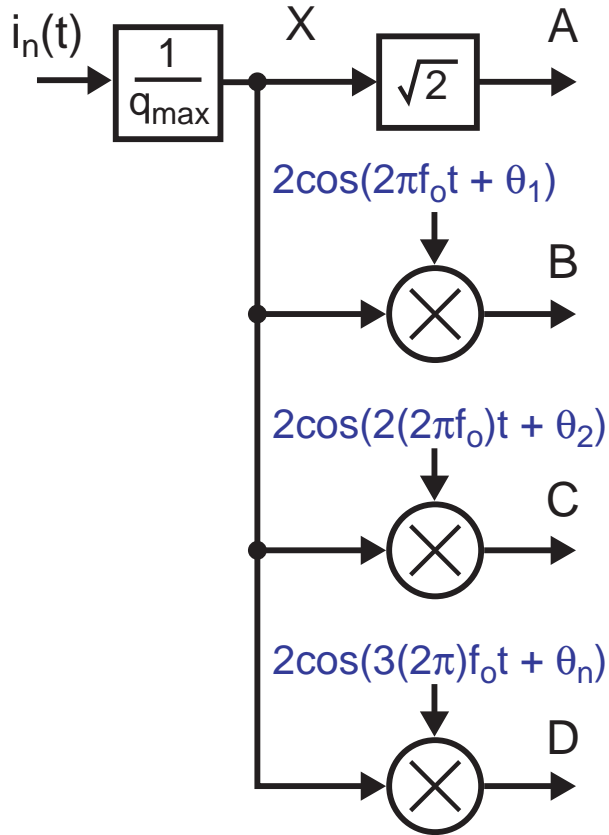
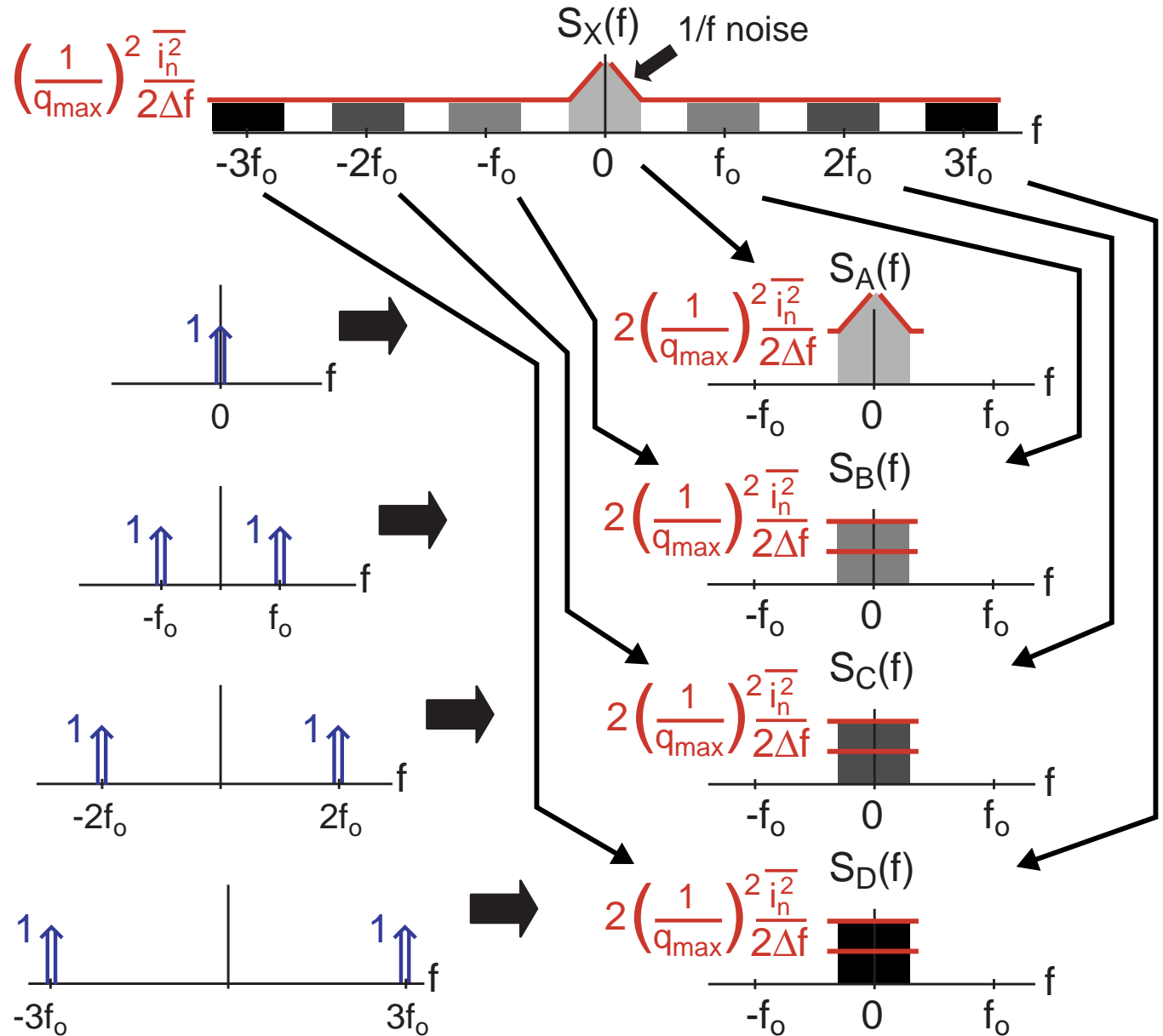
$$L(\Delta f) = 10 \log(S_{\Phi_{out}}(\Delta f))$$

- Resulting phase noise**

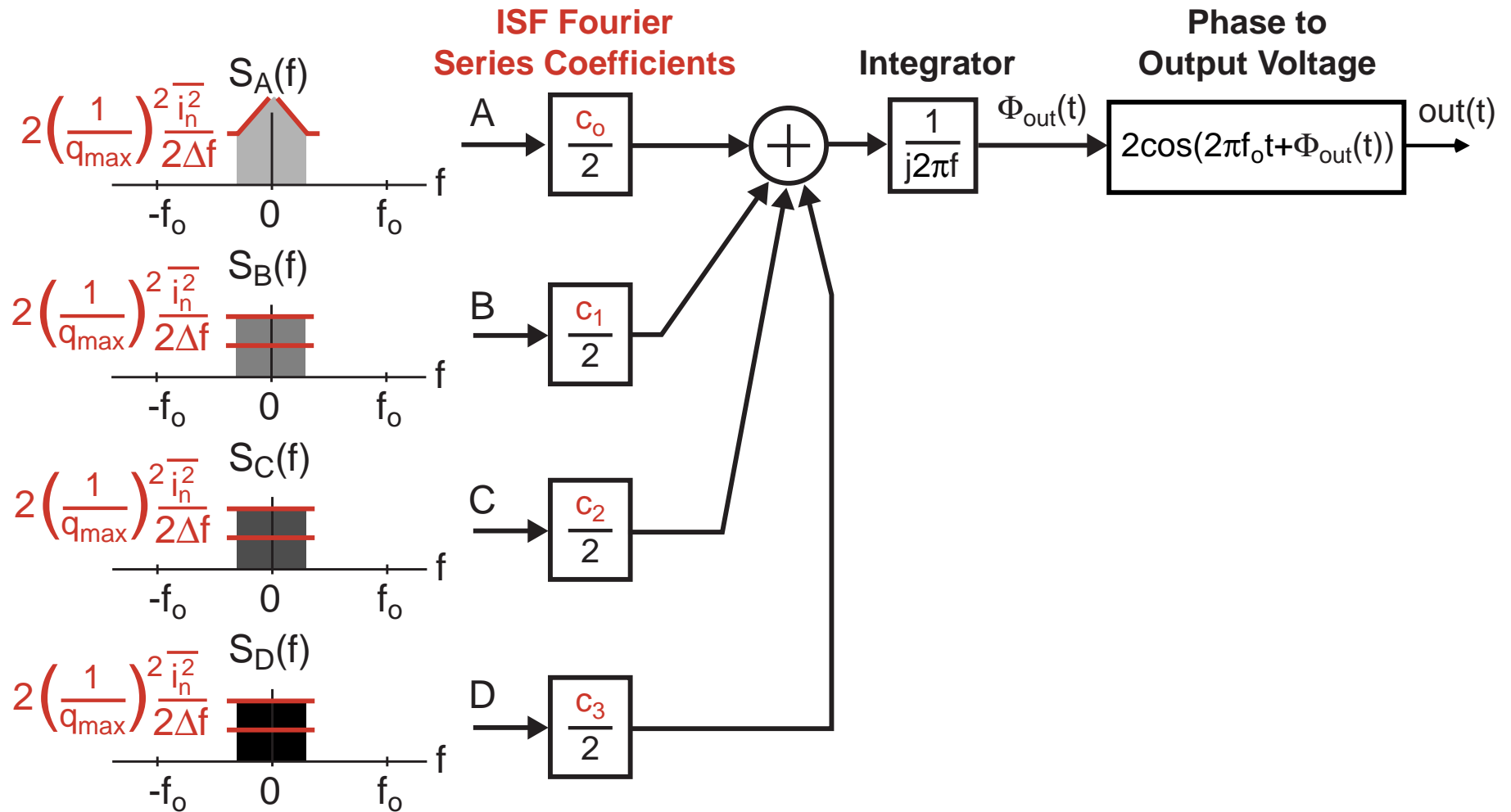
$$L(\Delta f) = 10 \log \left(\left(\frac{1}{2\pi \Delta f} \right)^2 \left(\sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left(\frac{1}{q_{max}} \right)^2 \frac{\overline{i_n^2}}{\Delta f} \right)$$

The Impact of 1/f Noise in Input Current (Part 1)

Note that $\frac{\overline{i_n^2}}{\Delta f}$ is the single-sided noise spectral density of $i_n(t)$



The Impact of 1/f Noise in Input Current (Part 2)



$$S_{\Phi_{out}}(f) \Big|_{1/f^3} = \left| \frac{1}{j2\pi f} \right|^2 \left(\frac{c_0}{2} \right)^2 S_A(f)$$

Calculation of Output Phase Noise in $1/f^3$ region

- From the previous slide

$$S_{\Phi_{out}}(f) \Big|_{1/f^3} = \left(\frac{1}{2\pi f} \right)^2 \left(\frac{c_o}{2} \right)^2 S_A(f)$$

- Assume that input current has $1/f$ noise with corner frequency $f_{1/f}$

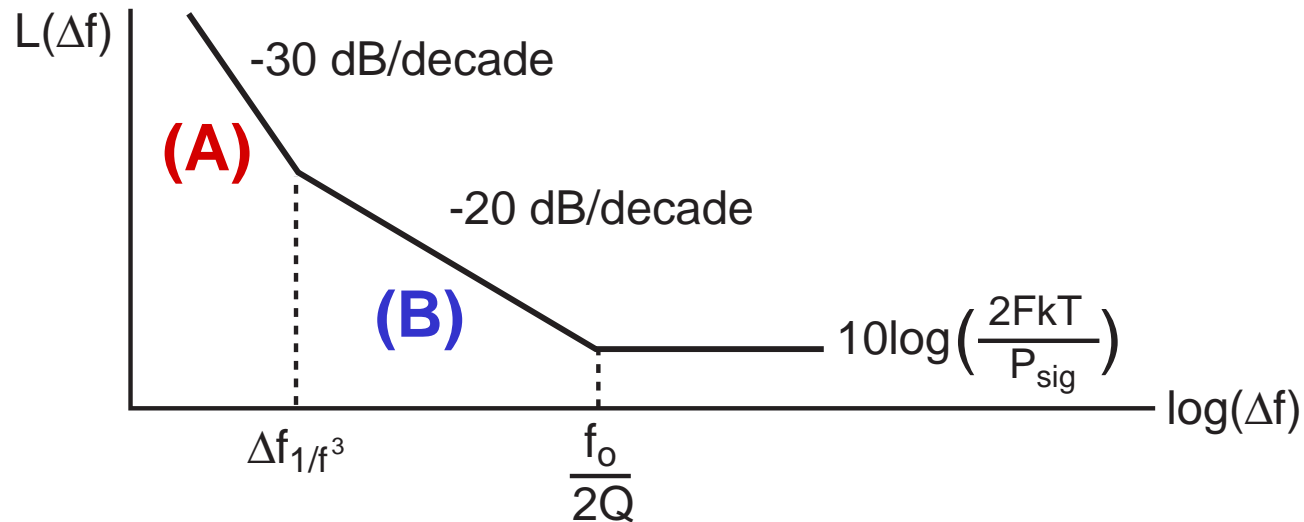
$$S_A(f) = \left(\frac{1}{q_{max}} \right)^2 \frac{\overline{i_n^2}}{\Delta f} \left(\frac{f_{1/f}}{\Delta f} \right)$$

- Corresponding output phase noise

$$L(\Delta f) \Big|_{1/f^3} = 10 \log \left(\left(\frac{1}{2\pi \Delta f} \right)^2 \left(\frac{c_o}{2} \right)^2 S_A(f) \right)$$

$$= 10 \log \left(\left(\frac{1}{2\pi \Delta f} \right)^2 (c_o^2) \frac{1}{4} \left(\frac{1}{q_{max}} \right)^2 \frac{\overline{i_n^2}}{\Delta f} \left(\frac{f_{1/f}}{\Delta f} \right) \right)$$

Calculation of $1/f^3$ Corner Frequency



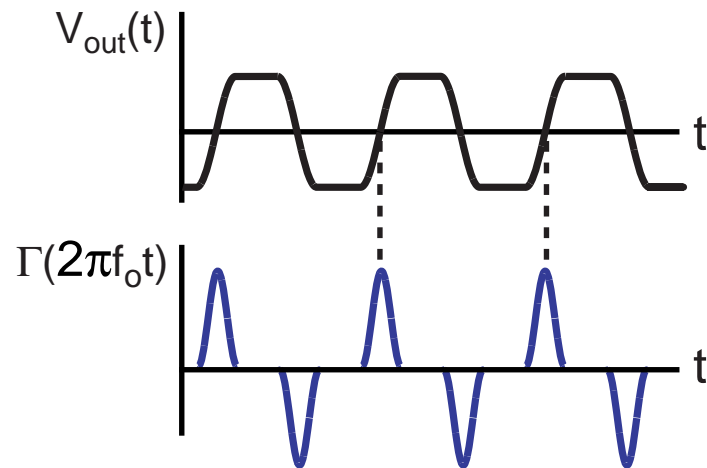
$$(A) \quad L(\Delta f) \Big|_{1/f^3} = 10 \log \left(\left(\frac{1}{2\pi \Delta f} \right)^2 (c_o^2) \frac{1}{4} \left(\frac{1}{q_{max}} \right)^2 \frac{\overline{i_n^2}}{\Delta f} \left(\frac{f_{1/f}}{\Delta f} \right) \right)$$

$$(B) \quad L(\Delta f) = 10 \log \left(\left(\frac{1}{2\pi \Delta f} \right)^2 \left(\sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left(\frac{1}{q_{max}} \right)^2 \frac{\overline{i_n^2}}{\Delta f} \right)$$

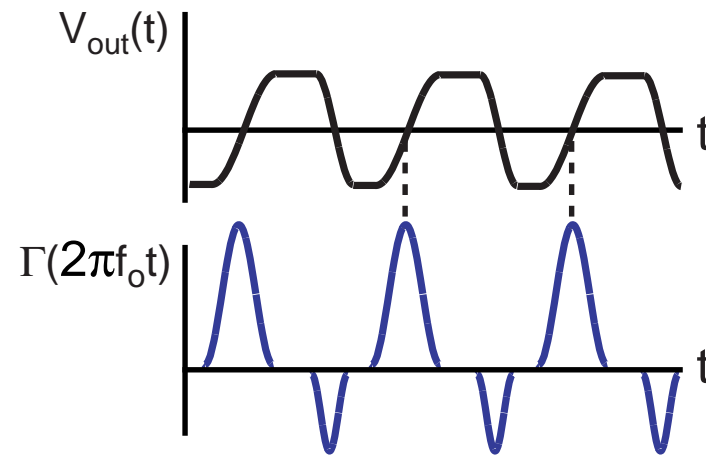
$$(A) = (B) \text{ at: } \Rightarrow \Delta f_{1/f^3} = \left(c_o^2 / \sum_{n=0}^{\infty} c_n^2 \right) f_{1/f}$$

Impact of Oscillator Waveform on $1/f^3$ Phase Noise

ISF for Symmetric Waveform

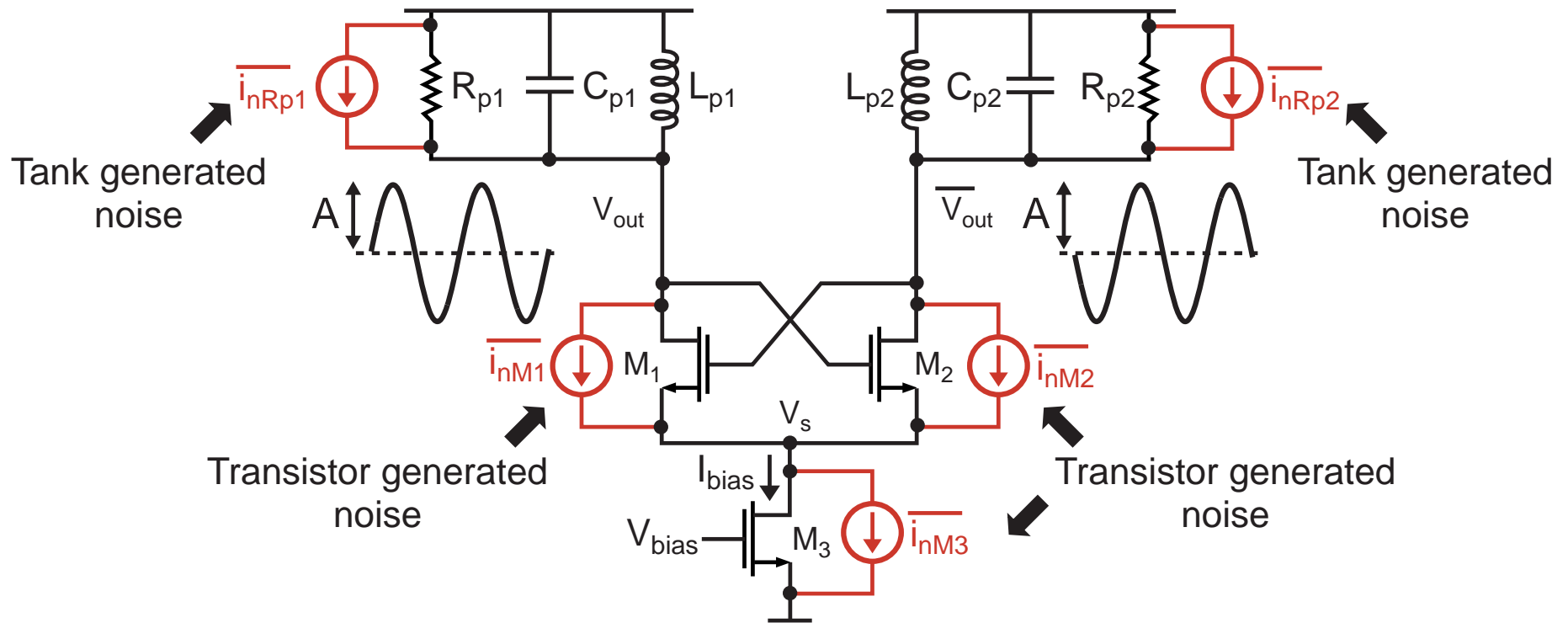


ISF for Asymmetric Waveform



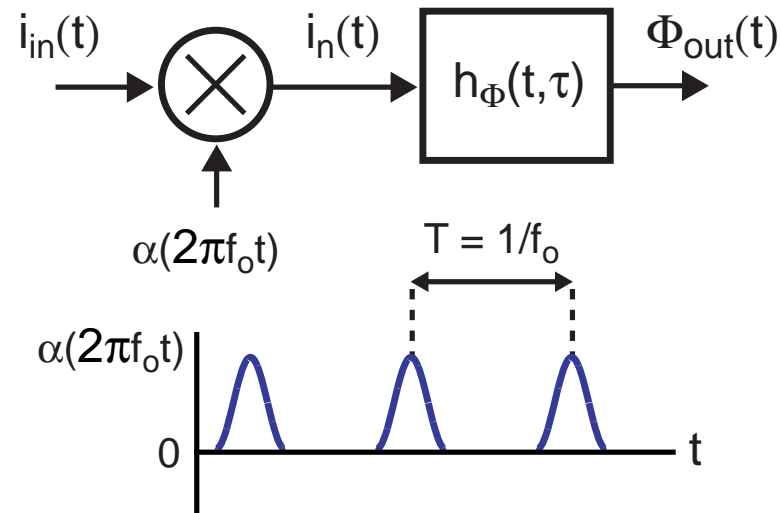
- **Key Fourier series coefficient of ISF for $1/f^3$ noise is c_0**
 - If DC value of ISF is zero, c_0 is also zero
- **For symmetric oscillator output waveform**
 - DC value of ISF is zero – no upconversion of flicker noise! (i.e. output phase noise does not have $1/f^3$ region)
- **For asymmetric oscillator output waveform**
 - DC value of ISF is nonzero – flicker noise has impact

Issue – We Have Ignored Modulation of Current Noise



- In practice, transistor generated noise is modulated by the varying bias conditions of its associated transistor
 - As transistor goes from saturation to triode to cutoff, its associated noise changes dramatically
- Can we include this issue in the LTV framework?

Inclusion of Current Noise Modulation



- **Recall**

$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_\Phi(t, \tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^t \Gamma(2\pi f_0 \tau) i_n(\tau) d\tau$$

- **By inspection of figure**

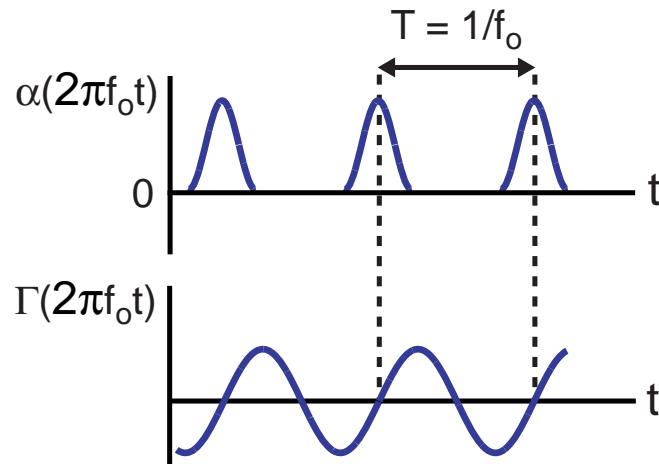
$$\Rightarrow \Phi_{out}(t) = \frac{1}{q_{max}} \int_{-\infty}^t \Gamma(2\pi f_0 \tau) \alpha(2\pi f_0 \tau) i_{in}(\tau) d\tau$$

- **We therefore apply previous framework with ISF as**

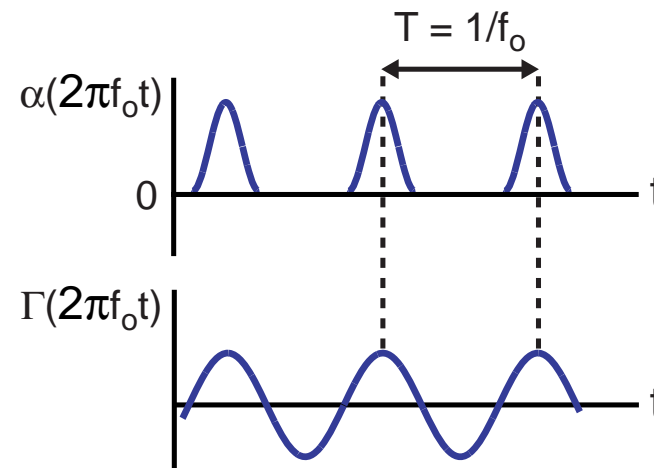
$$\Gamma_{eff}(2\pi f_0 \tau) = \Gamma(2\pi f_0 \tau) \alpha(2\pi f_0 \tau)$$

Placement of Current Modulation for Best Phase Noise

Best Placement of Current Modulation for Phase Noise



Worst Placement of Current Modulation for Phase Noise

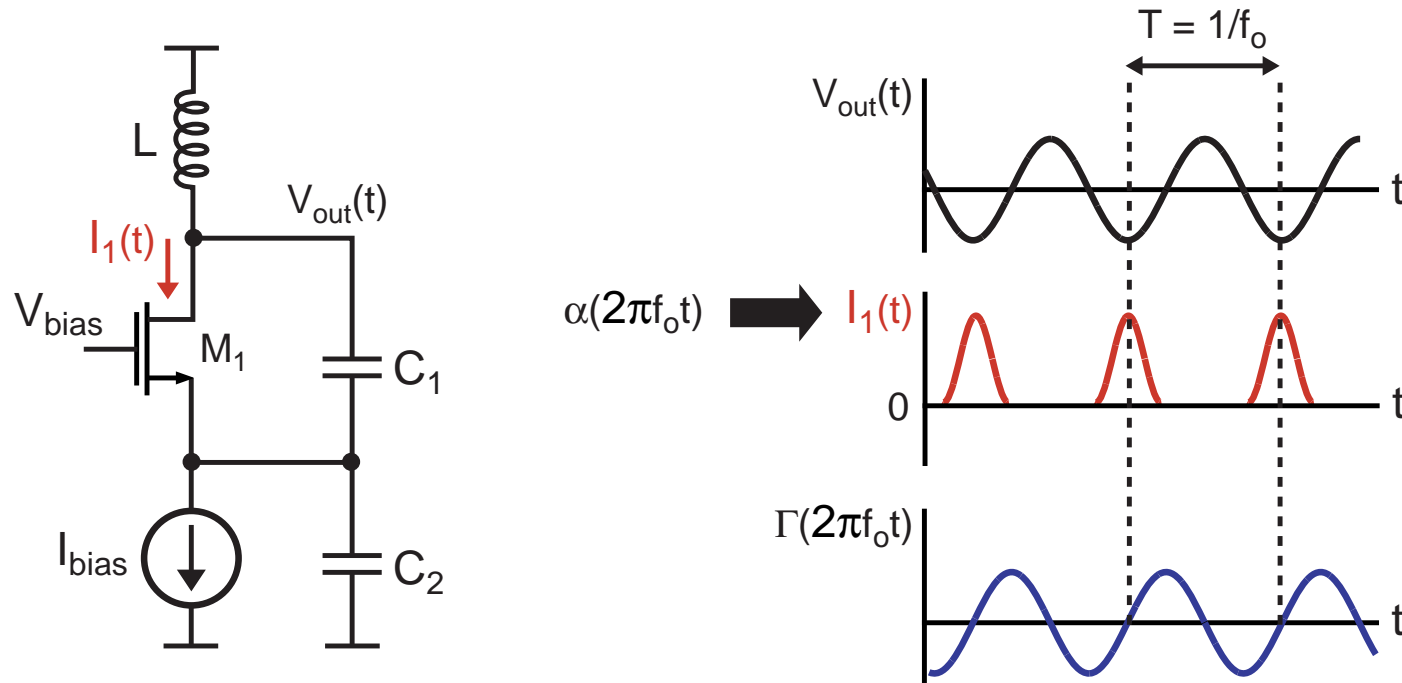


- Phase noise expression (ignoring $1/f$ noise)

$$L(\Delta f) = 10 \log \left(\left(\frac{1}{2\pi \Delta f} \right)^2 \left(\sum_{n=0}^{\infty} c_n^2 \right) \frac{1}{4} \left(\frac{1}{q_{max}} \right)^2 \frac{\overline{i_n^2}}{\Delta f} \right)$$

- Minimum phase noise achieved by minimizing sum of square of Fourier series coefficients (i.e. rms value of Γ_{eff})

Colpitts Oscillator Provides Optimal Placement of α

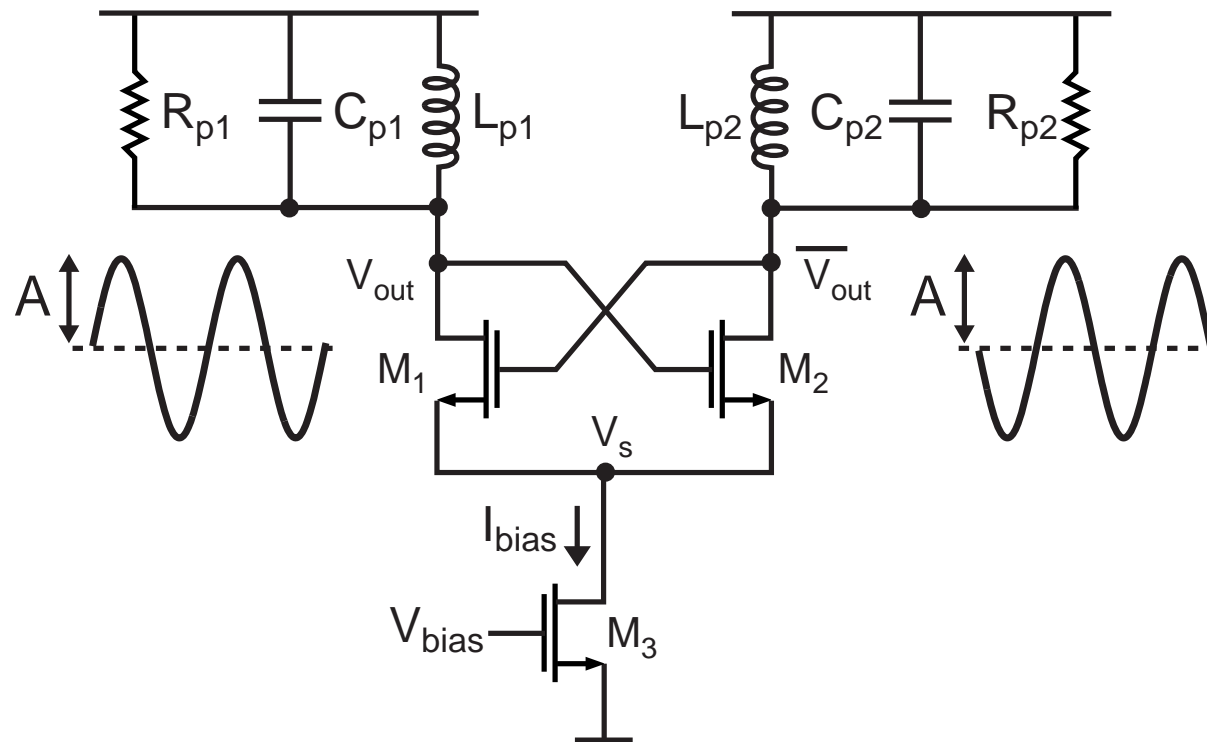


- **Current is injected into tank at bottom portion of VCO swing**
 - **Current noise accompanying current has minimal impact on VCO output phase**

Summary of LTV Phase Noise Analysis Method

- Step 1: calculate the impulse sensitivity function of each oscillator noise source using a simulator
- Step 2: calculate the noise current modulation waveform for each oscillator noise source using a simulator
- Step 3: combine above results to obtain $\Gamma_{\text{eff}}(2\pi f_o t)$ for each oscillator noise source
- Step 4: calculate Fourier series coefficients for each $\Gamma_{\text{eff}}(2\pi f_o t)$
- Step 5: calculate spectral density of each oscillator noise source (before modulation)
- Step 6: calculate overall output phase noise using the results from Step 4 and 5 and the phase noise expressions derived in this lecture (or the book)

$F(\Delta f)$ Has Been Determined for This Topology



- Rael et. al. have come up with a closed form expression for $F(\Delta f)$ for the above topology
- In the region where phase noise falls at -20 dB/dec:

$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{do, M3} R_p \quad (R_p = R_{p1} = R_{p2})$$

References to Rael Work

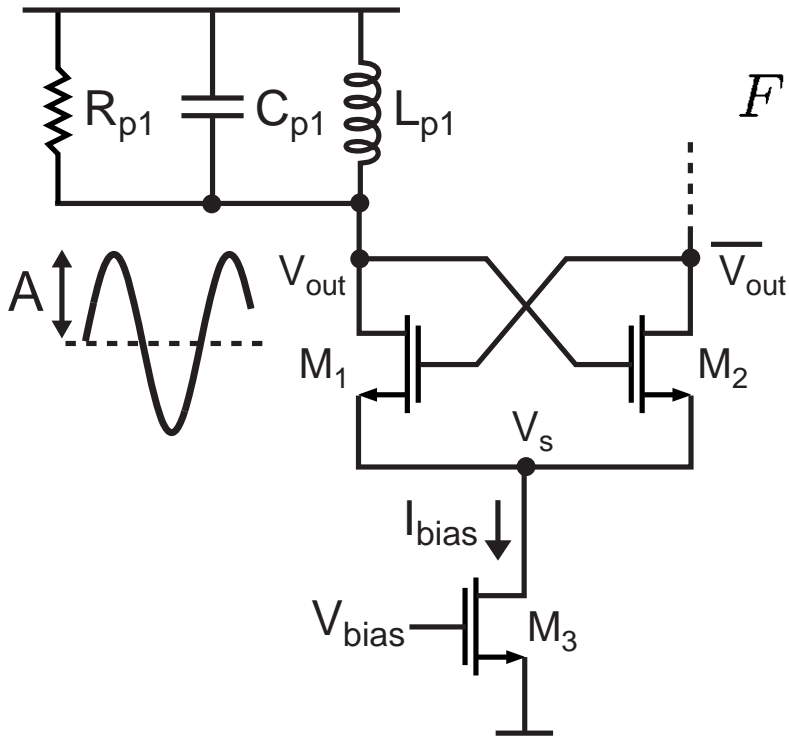
- **Phase noise analysis**

- J.J. Rael and A.A. Abidi, “Physical Processes of Phase Noise in Differential LC Oscillators”, Custom Integrated Circuits Conference, 2000, pp 569-572

- **Implementation**

- Emad Hegazi et. al., “A Filtering Technique to Lower LC Oscillator Phase Noise”, JSSC, Dec 2001, pp 1921-1930

Designing for Minimum Phase Noise



$$F(\Delta f) = \underbrace{1}_{(A)} + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)} + \underbrace{\frac{4}{9} g_{do, M3} R_p}_{(C)}$$

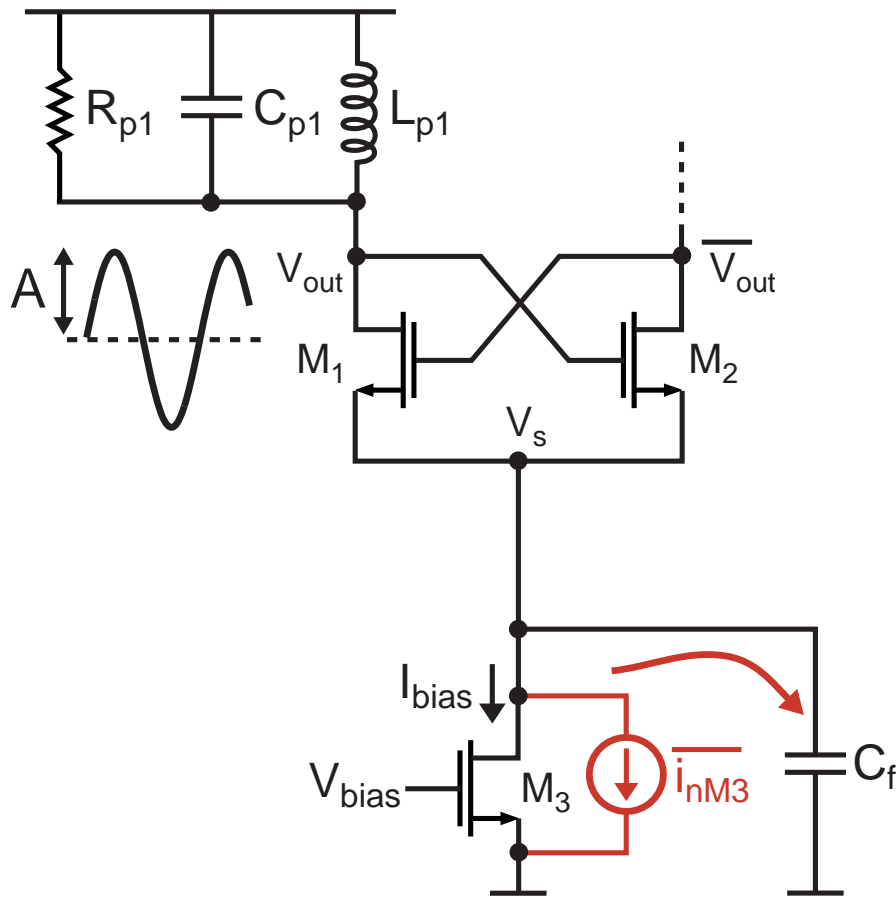
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

(C) Noise from M_3

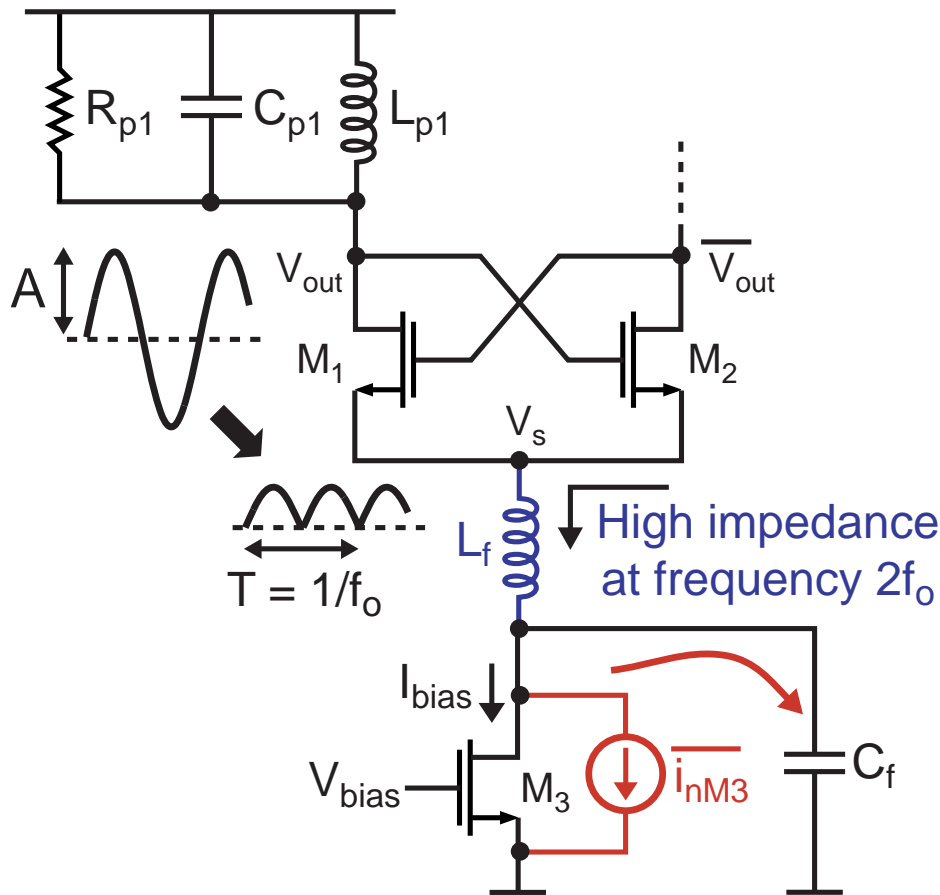
- To achieve minimum phase noise, we'd like to minimize $F(\Delta f)$
- The above formulation provides insight of how to do this
 - Key observation: **(C)** is often quite significant

Elimination of Component (C) in $F(\Delta f)$



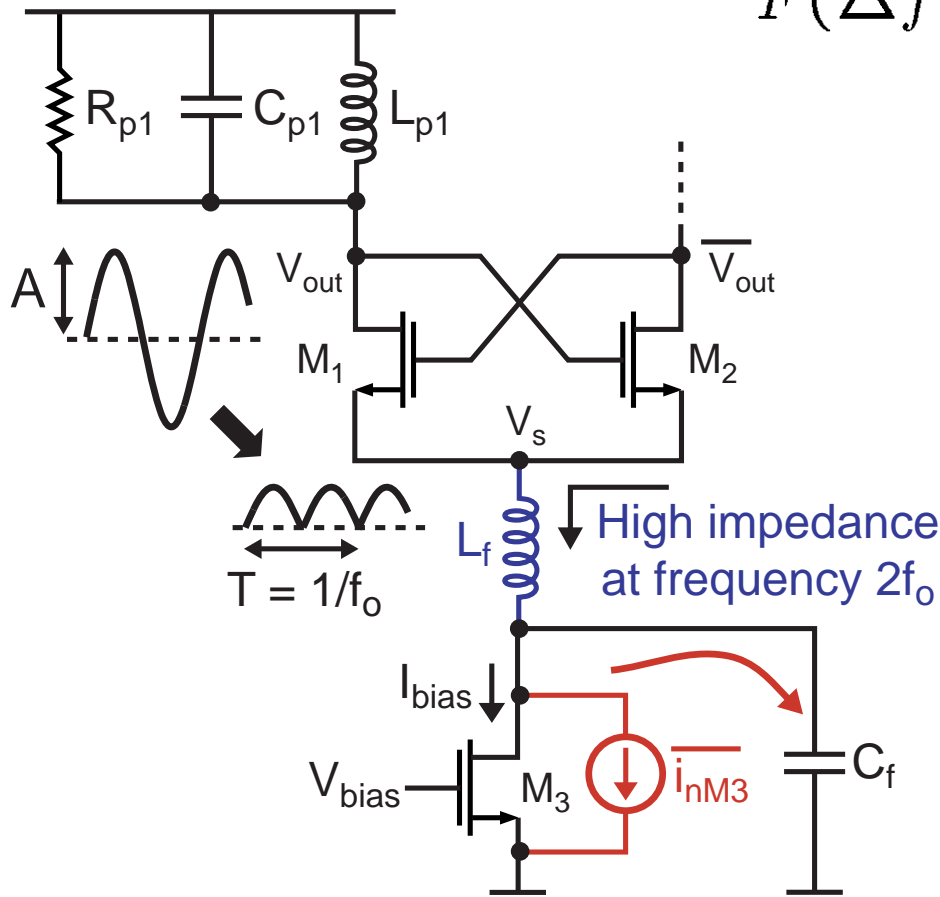
- Capacitor C_f shunts noise from M_3 away from tank
 - Component (C) is eliminated!
- Issue – impedance at node V_s is very low
 - Causes M_1 and M_2 to present a low impedance to tank during portions of the VCO cycle
 - Q of tank is degraded

Use Inductor to Increase Impedance at Node V_s



- Voltage at node V_s is a rectified version of oscillator output
 - Fundamental component is at twice the oscillation frequency
- Place inductor between V_s and current source
 - Choose value to resonate with C_f and parasitic source capacitance at frequency $2f_o$
- Impedance of tank not degraded by M_1 and M_2
 - Q preserved!

Designing for Minimum Phase Noise – Next Part



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{m3} R_p$$

(A)
(B)
(C)

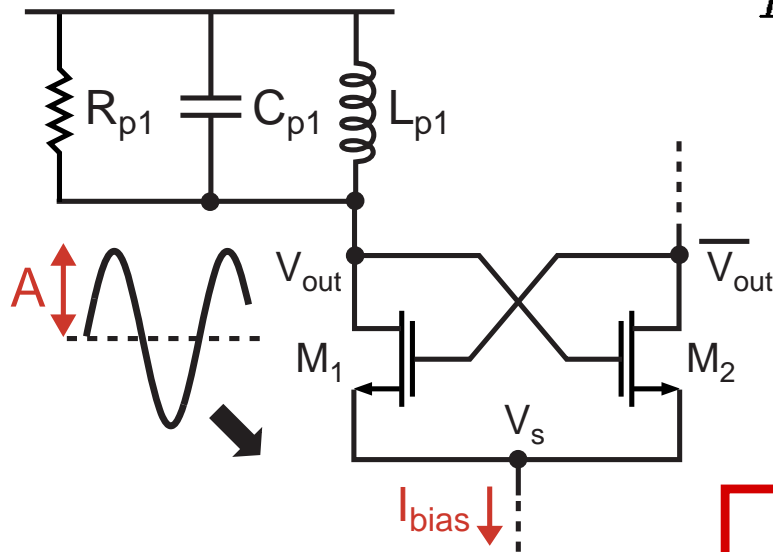
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

~~(C) Noise from M_3~~

- Let's now focus on component (B)
 - Depends on bias current and oscillation amplitude

Minimization of Component (B) in $F(\Delta f)$



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A}$$

(B)

■ **Recall from Lecture 11**

$$A = \frac{2}{\pi} I_{bias} R_p$$

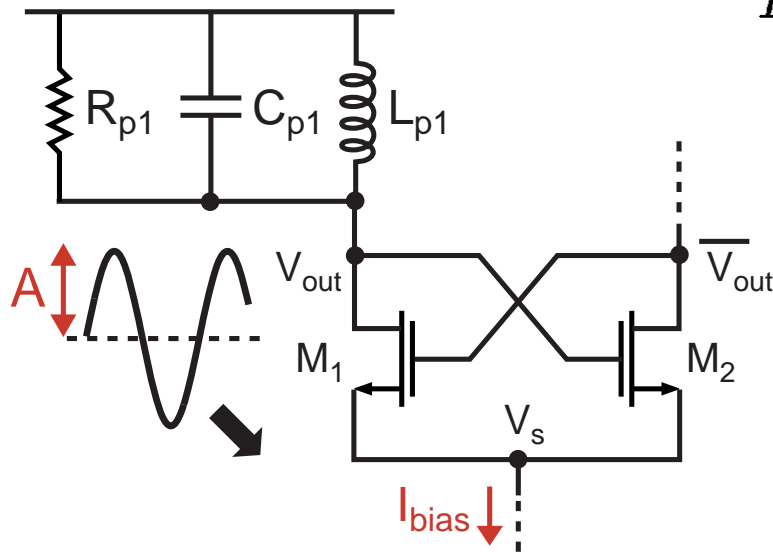
$$\Rightarrow F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi (2/\pi) I_{bias} R_p} = 1 + \gamma$$

■ **So, it would seem that I_{bias} has no effect!**

- **Not true – want to maximize A (i.e. P_{sig}) to get best phase noise, as seen by:**

$$L(\Delta f) = 10 \log \left(\frac{2kTF(\Delta f)}{P_{sig}} \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right)$$

Current-Limited Versus Voltage-Limited Regimes



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} \quad \text{(B)}$$

- Oscillation amplitude, A , cannot be increased above supply imposed limits
- If I_{bias} is increased above the point that A saturates, then (B) increases

- Current-limited regime: amplitude given by $A = \frac{2}{\pi} I_{bias} R_p$
- Voltage-limited regime: amplitude saturated

Best phase noise achieved at boundary between these regimes!

Final Comments

- **Hajimiri method useful as a numerical procedure to determine phase noise**
 - Provides insights into $1/f$ noise upconversion and impact of noise current modulation
- **Rael method useful for CMOS negative-resistance topology**
 - Closed form solution of phase noise!
 - Provides a great deal of design insight
- **Another numerical method**
 - Spectre RF from Cadence now does a reasonable job of estimating phase noise for many oscillators
 - Useful for verifying design ideas and calculations