Analysis and Design of Analog Integrated Circuits
Lecture 9

Open Circuit Time Constant Technique

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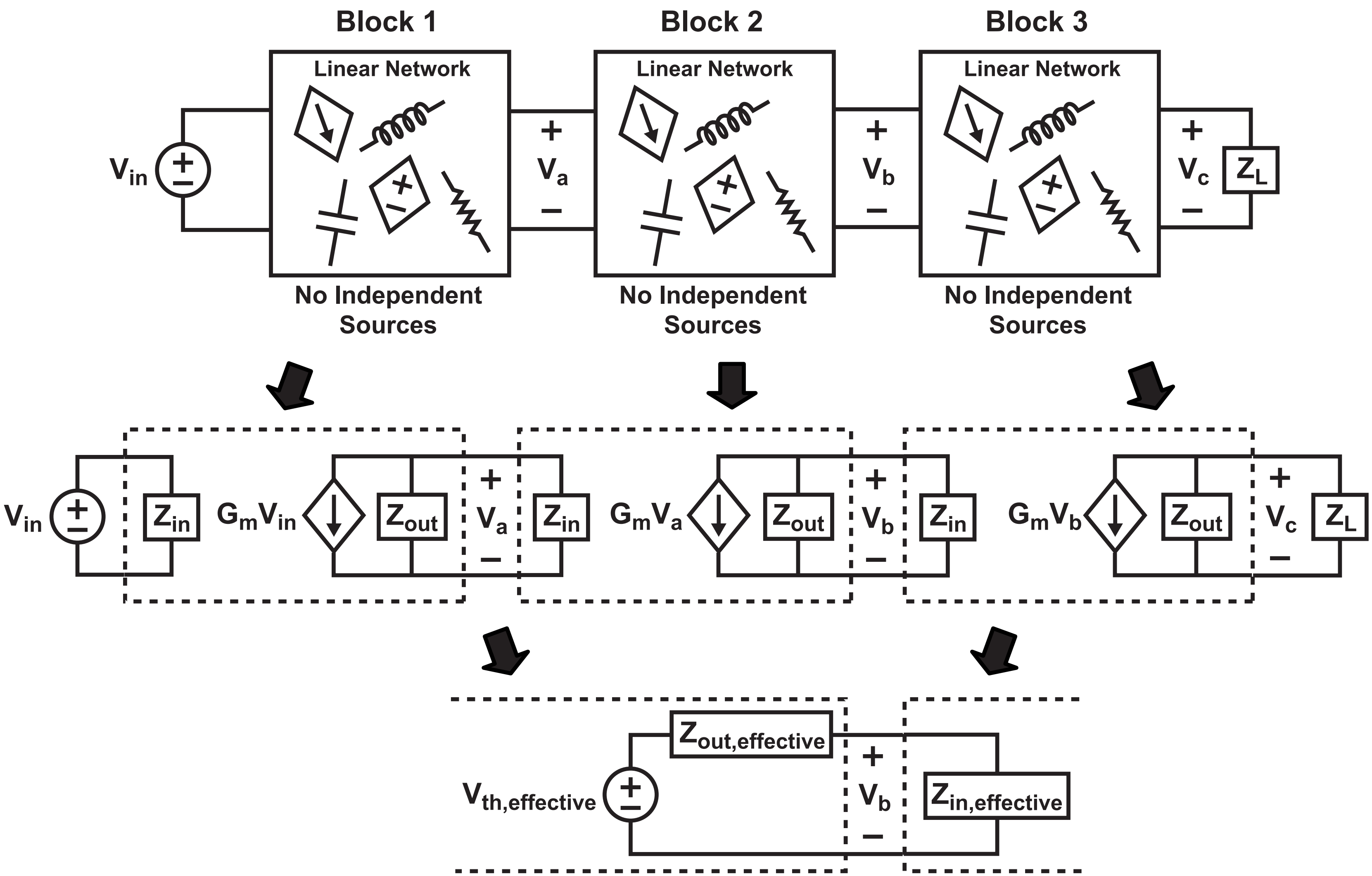
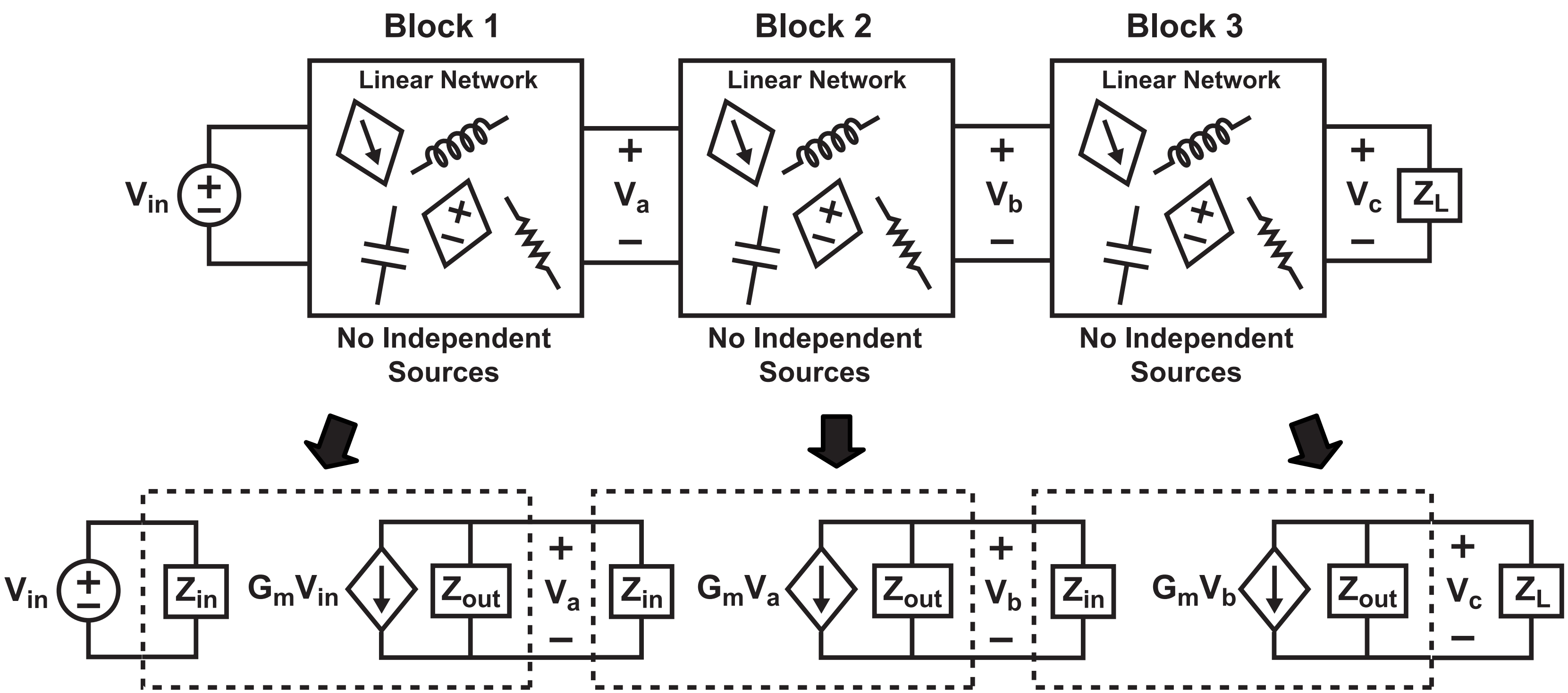
Review of Our Analysis Techniques



- Two port analysis allows us to quickly calculate small signal gain from cascaded network stages
  - So far, only purely resistive impedances have been considered

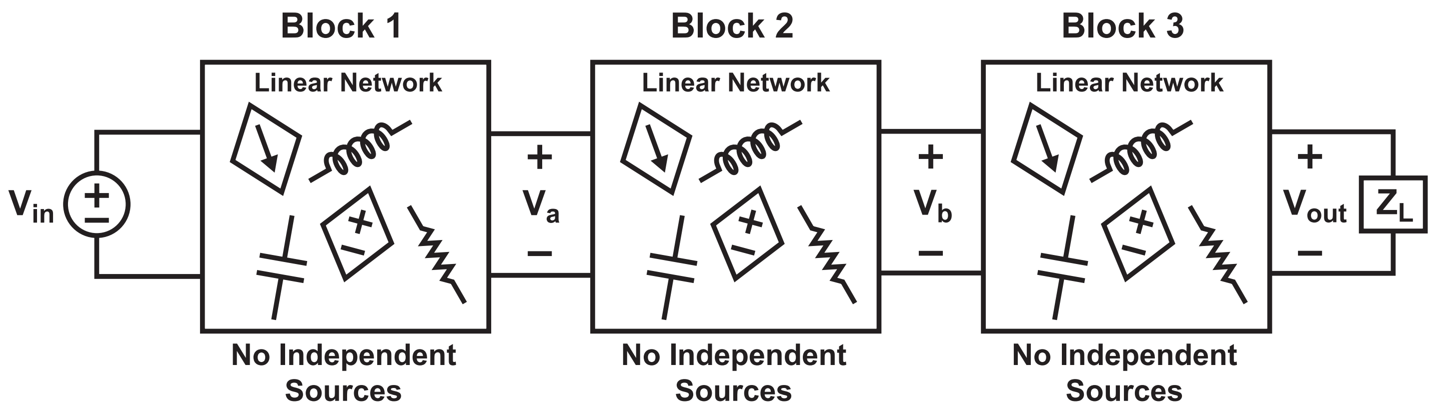
The Problem with Complex Impedances

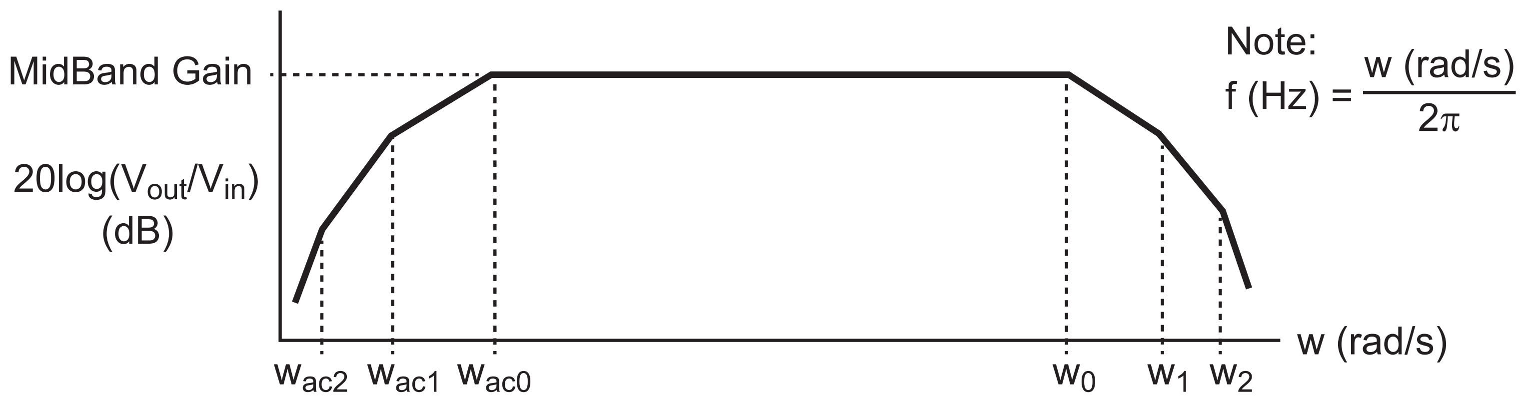


- When complex impedances are considered (i.e., capacitors, inductors, and resistors), things get much more messy
  - Complex impedance calculations are time consuming
  - Capacitance between drain and gate of transistors complicates calculation effort further

Can we determine a faster analysis path to gain intuition?

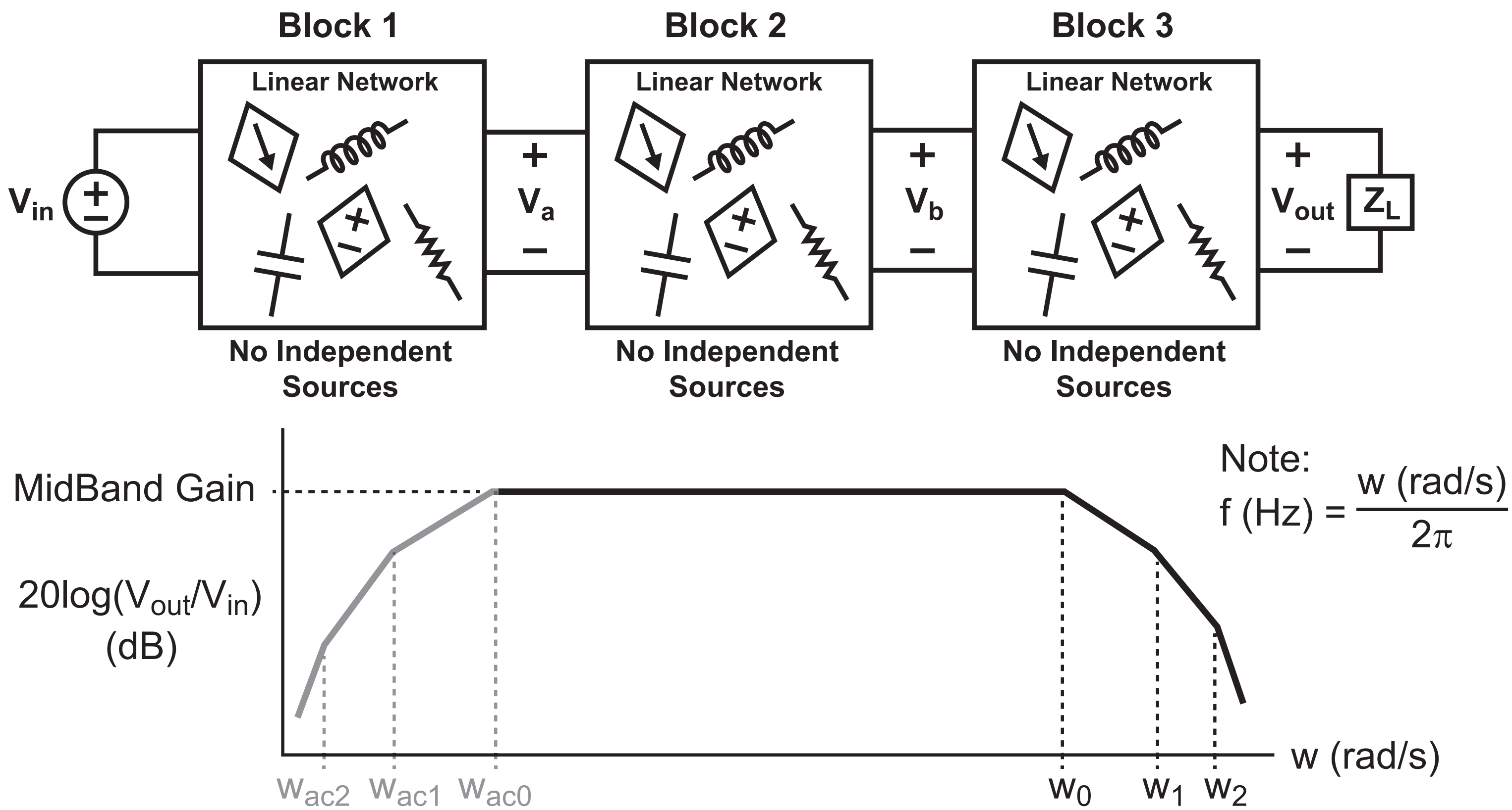
General Frequency Response for Amplifiers





- **Midband gain can be calculated by assuming purely resistive impedances (as we have done so far)**
  - **Large valued capacitors used for AC coupling will be shorts in this analysis**
    - For DC coupled circuits, typically DC gain = Midband Gain
  - **Small valued capacitors will be opens in this analysis**

Our Focus Will Be on High Frequency Poles

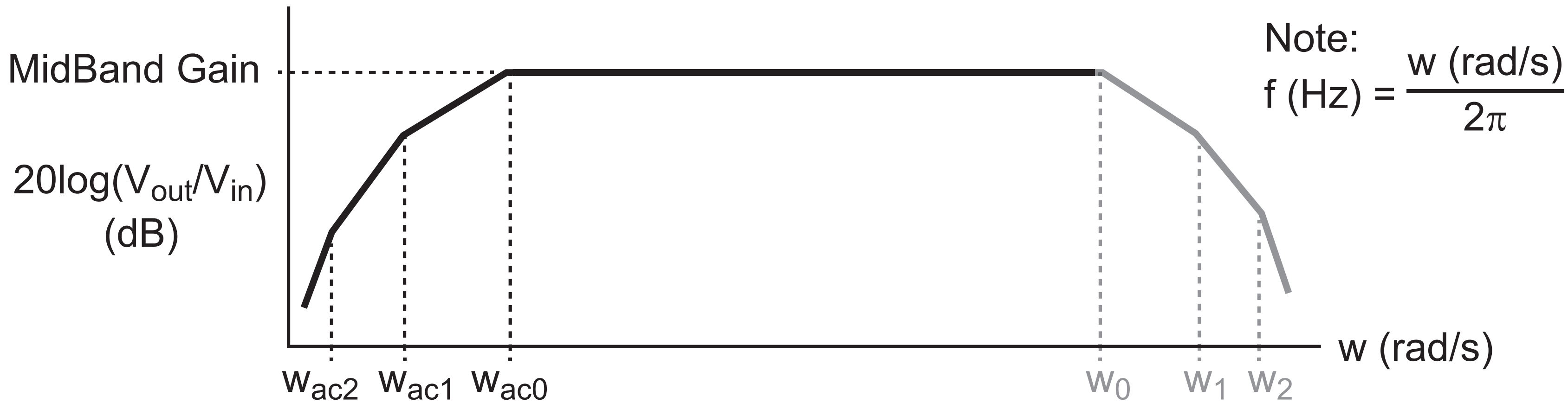


- We are particularly interested in knowing the bandwidth of our amplifier circuit
  - Bandwidth is primarily set by the lowest frequency pole, w_0
  - Additional attenuation occurs at frequencies beyond the amplifier bandwidth by higher frequency poles w_1, w_2, etc.

Open Circuit Time Constant Technique

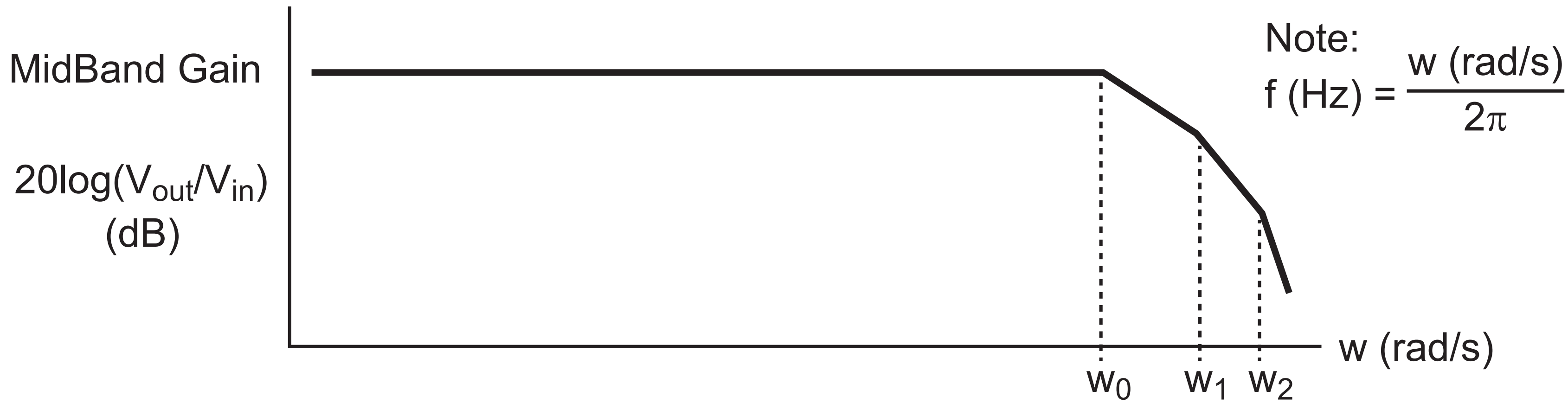
- **The Open Circuit Time Constant (OCT) technique allows us to quickly estimate the bandwidth of an amplifier circuit**
  - **We will see that it is most accurate when there is one *dominant* pole, w_0**
    - This means that w_1, w_2, and higher poles are not close in frequency to w_0
      - This will hold for opamps and other circuits that operate in feedback
    - There is still considerable value to the OCT method in providing design intuition even when there is not just one dominant pole

Short Circuit Time Constant Technique



- **The Short Circuit Time Constant (SCT) technique allows us to quickly estimate the AC-coupled cutoff frequency, w_{ac0}**
  - **This has many similarities to the OCT method, but we will not discuss in this class since**
    - AC coupling is not used very often in integrated circuits due to the high cost of large valued capacitors
    - When AC coupling is applied in integrated circuits, it is often quite easy to estimate the AC-coupled cutoff frequency since there are relatively few poles in the circuit related to AC-coupling

Key Assumptions for the OCT Technique

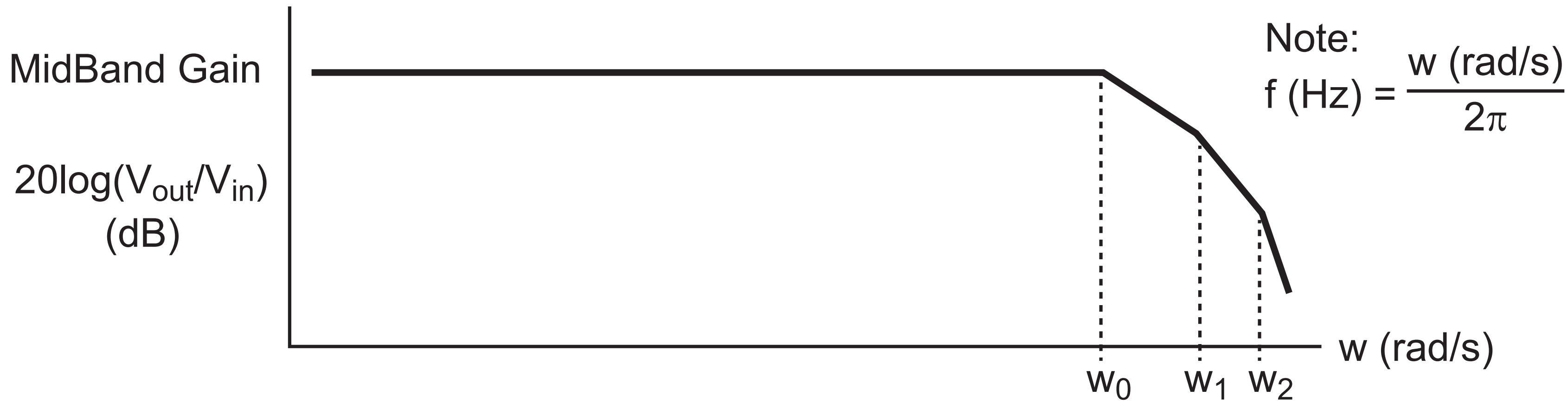


- **Let us assume that the transfer function from V_{in} to V_{out} is**

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{(\tau_0 s + 1)(\tau_1 s + 1)\cdots(\tau_{n-1} s + 1)}$$

  - **Note that we are ignoring any AC-coupling poles/zeros**
    - This implies that are approximating DC gain = Midband gain
    - The OCT method does not require this assumption – it just simplifies the analysis to follow
  - **Note also that DC gain equals *K* in the above transfer function**
    - We see this by setting *s* = 0

Key Idea of the OCT Technique



- **Assuming the transfer function from V_{in} to V_{out} is:**

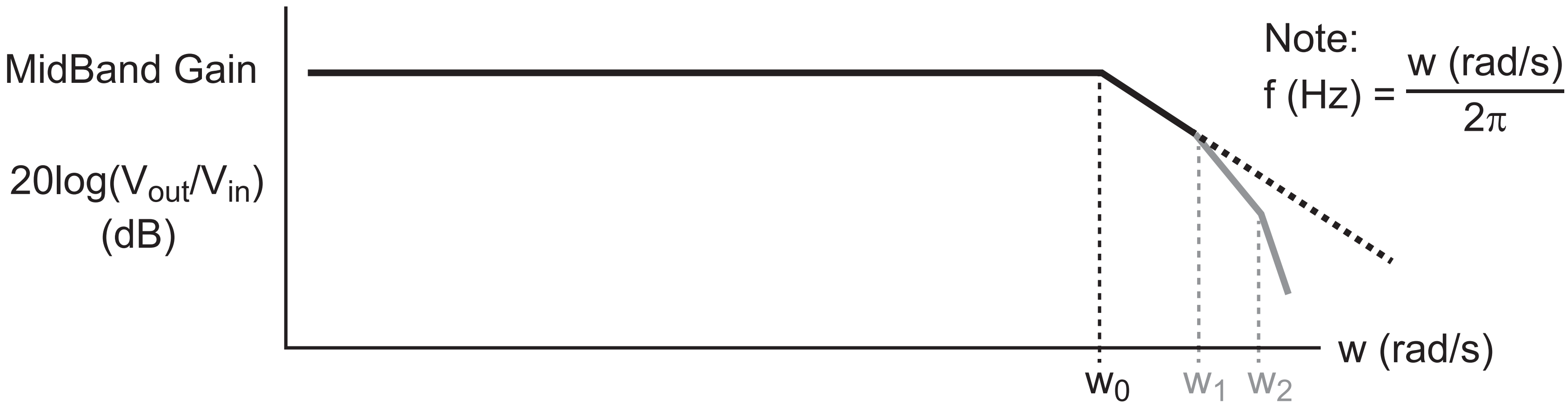
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{(\tau_0 s + 1)(\tau_1 s + 1)\cdots(\tau_{n-1} s + 1)}$$

- **We can achieve a reasonable approximation of the bandwidth of the system by instead considering:**

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{\left(\sum_{i=0}^{n-1} \tau_i\right) s + 1}$$

  - **Here τ_i are the "time constants" corresponding to the poles of the circuit network**

Bandwidth Estimate from OCT Technique



- **The OCT technique approximates the transfer function as:**

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{\left(\sum_{i=0}^{n-1} \tau_i\right) s + 1}$$

- **The estimated bandwidth is found by substituting $s = jw_0$ and solving for w_0 such that the magnitude is $K/\sqrt{2}$**

$$w_0 = \frac{1}{\sum_{i=0}^{n-1} \tau_i} \Rightarrow \left|\frac{V_{out}(w_0)}{V_{in}(w_0)}\right| = \left|\frac{K}{j1+1}\right| = \frac{K}{\sqrt{2}}$$

Bandwidth estimate found by inversing the sum of time constants!

Why Is This Approximation Reasonable?

- **Consider a second order example:**

$$\frac{V_{out}(w_0)}{V_{in}(w_0)} = \frac{K}{(j\tau_0 w_0 + 1)(j\tau_1 w_0 + 1)}$$

- **Expanding:**

$$\frac{V_{out}(w_0)}{V_{in}(w_0)} = \frac{K}{-\tau_0\tau_1 w_0^2 + j(\tau_0 + \tau_1)w_0 + 1}$$

  - **But notice (since the time constant values are > 0):**

$$j(\tau_0 + \tau_1)w_0 = j1 \ \Rightarrow \ \tau_0 w_0 < 1, \ \tau_1 w_0 < 1$$

    - In fact: $\tau_0\tau_1 w_0^2 \leq 0.25$
    - The worse case of $\tau_0\tau_1\omega_o^2 = 0.25$ occurs when $\tau_0 = \tau_1$:

$$\left|\frac{V_{out}(w_0)}{V_{in}(w_0)}\right| = \left|\frac{K}{j1 + 1 - 0.25}\right| = \frac{K}{\sqrt{1.56}} \approx \frac{K}{\sqrt{2}}$$

    - The approximation will be better for $\tau_o \neq \tau_1$

Key Issues For the OCT Approximation

- **For the higher order transfer function**

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K}{(\tau_0 s + 1)(\tau_1 s + 1)\cdots(\tau_{n-1} s + 1)}$$

- **The OCT approximation for bandwidth is**

$$BW \approx \frac{1}{\sum_{i=0}^{n-1} \tau_i} \; rad/s$$

- **As hinted at by our second order example:**
  - **The OCT approximation will have much better accuracy if the time constants are different, and particularly if there is one dominant time constant**
  - **The bandwidth estimate by the OCT method is typically conservative (i.e., actual bandwidth > OCT estimate)**
    - Complex poles can lead to actual bandwidth < OCT estimate

But how do we compute $\sum_{i=0}^{n-1} \tau_i$?

OCT Method of Calculating the Sum of Time Constants

- **OCT method calculates $\sum_{i=0}^{n-1} \tau_i$ by the following steps:**
  - **Compute the effective resistance R_{thj} seen by each capacitor, C_j, with other caps as open circuits**
    - AC coupling caps are not included – considered as shorts
  - **Form the "open circuit" time constant $T_j = R_{thj}C_j$ for each capacitor C_j**
  - **Sum all of the "open circuit" time constants**
- **As proved by Richard Adler at MIT**

$$\sum_{i=0}^{n-1} \tau_i = \sum_{j=1}^{m} R_{thj} C_j$$

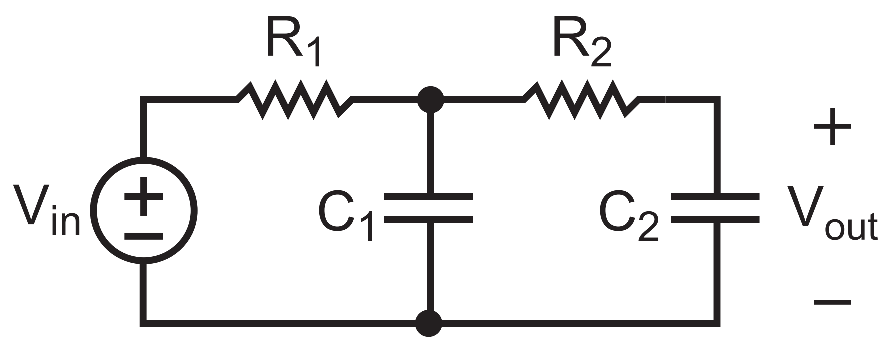
  - **This implies that the sum of the transfer function pole time constants is the same as the sum of the open circuit time constants**

$$\Rightarrow \; BW \approx \frac{1}{\sum_{j=1}^{m} R_{thj} C_j} \; rad/s$$

How Do You Tell if a Cap is for AC coupling or OCT?

- **In general, capacitors associated with AC coupling have the property that the amplifier gain *increases* as the capacitor goes from open to short**
  - **These capacitors are simply assumed to be shorts for the OCT analysis**
- **In general, capacitors used in the OCT calculation have the property that the amplifier gain *decreases* as the capacitor goes from open to short**
  - **These capacitors must all be considered in the OCT analysis**

Example: Second Order RC Network



- **Transfer function of the above network:**

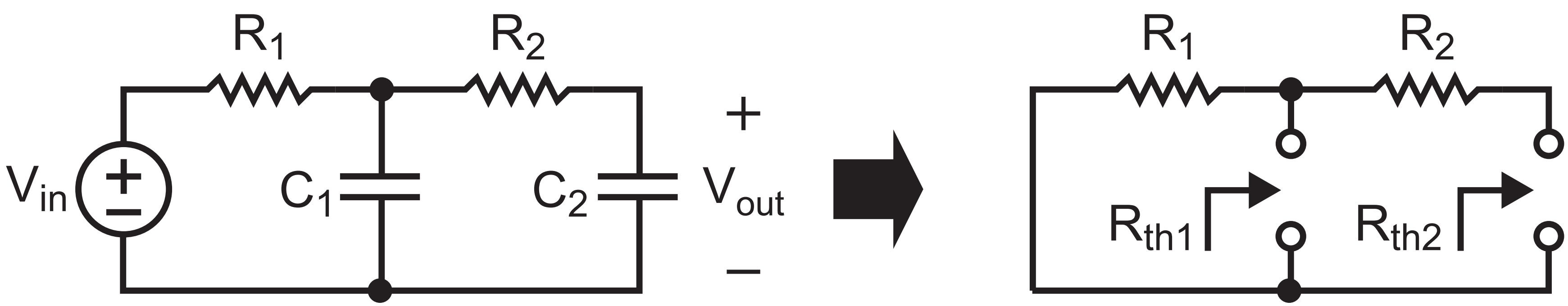
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2)s + 1}$$

- **The sum of the time constants from the poles of the above network are obtained by inspection of the first order coefficient in the above transfer function**

$$\Rightarrow \; BW \approx \frac{1}{\sum_{i=0}^{n-1} \tau_i} = \frac{1}{R_1 C_1 + R_1 C_2 + R_2 C_2} \; rad/s$$

- **For more complex networks, the direct approach of explicitly calculating the transfer function is quite tedious**

OCT Method Applied to Second Order RC Network



- **Obtain the Thevenin resistance values seen by each capacitor with other capacitors as opens**

$$R_{th1} = R_1 \;\Rightarrow\; R_{th1}C_1 = R_1C_1$$

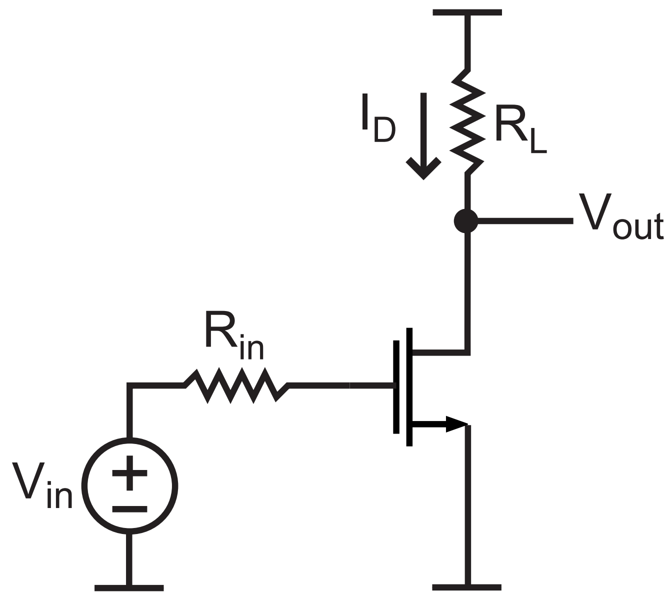
$$R_{th2} = R_1 + R_2 \;\Rightarrow\; R_{th2}C_2 = (R_1 + R_2)C_2$$

- **Bandwidth estimate from OCT method:**

$$\Rightarrow\; BW \approx \frac{1}{\sum_{j=1}^{m} R_{thj}C_j} = \frac{1}{R_1C_1 + (R_1 + R_2)C_2}\;\; rad/s$$

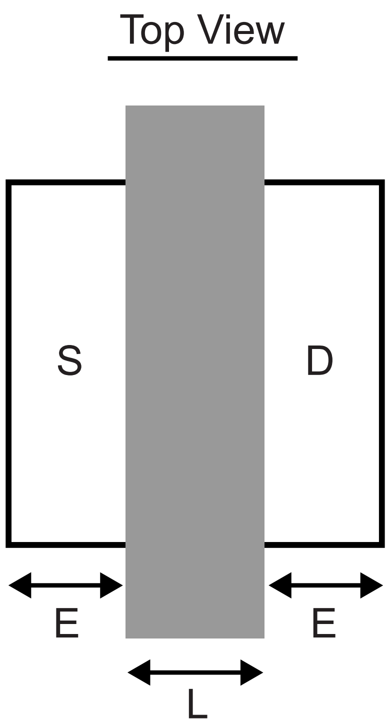
  - **Note that OCT method agrees with estimate based on direct calculation of the transfer function, but is much faster!**

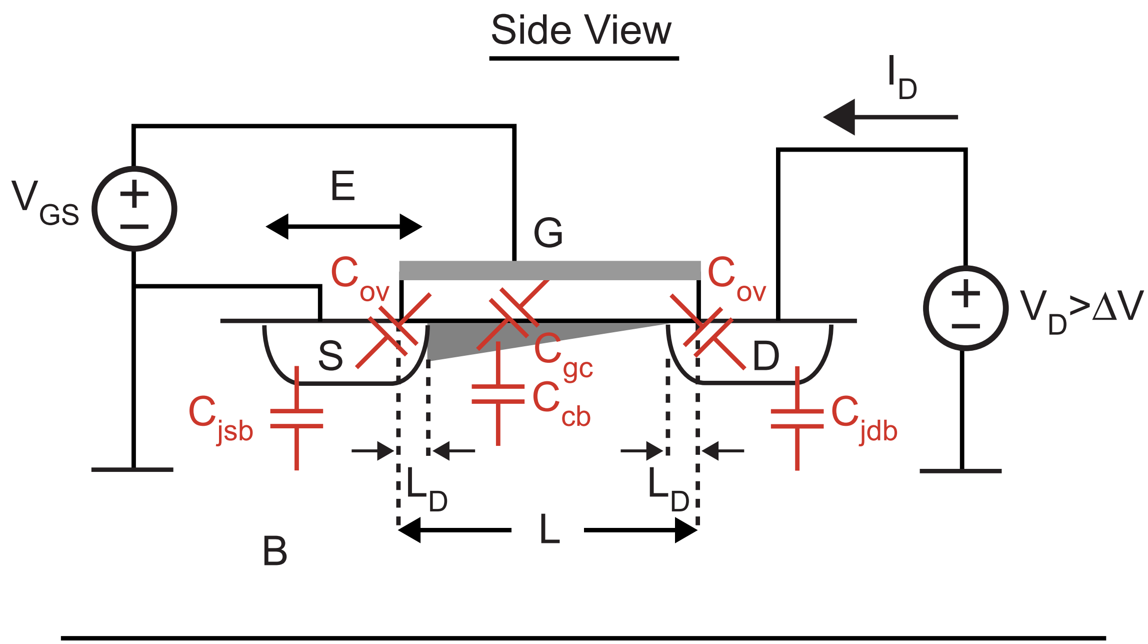
Example: Common Source Amplifier



- **Estimate the bandwidth of the above amplifier using the OCT method**
  - **What capacitances should be considered?**
  - **What Thevenin resistances must be calculated?**

Key Capacitances for CMOS Devices





junction bottom wall cap (per area)

junction sidewall cap (per length)

source to bulk cap: $C_{jsb} = \dfrac{C_j(0)}{\sqrt{1 + V_{SB}/|\Phi_B|}} WE + \dfrac{C_{jsw}(0)}{\sqrt{1 + V_{SB}/|\Phi_B|}} (W + 2E)$

(make 2W for "4 sided" perimeter in some cases)

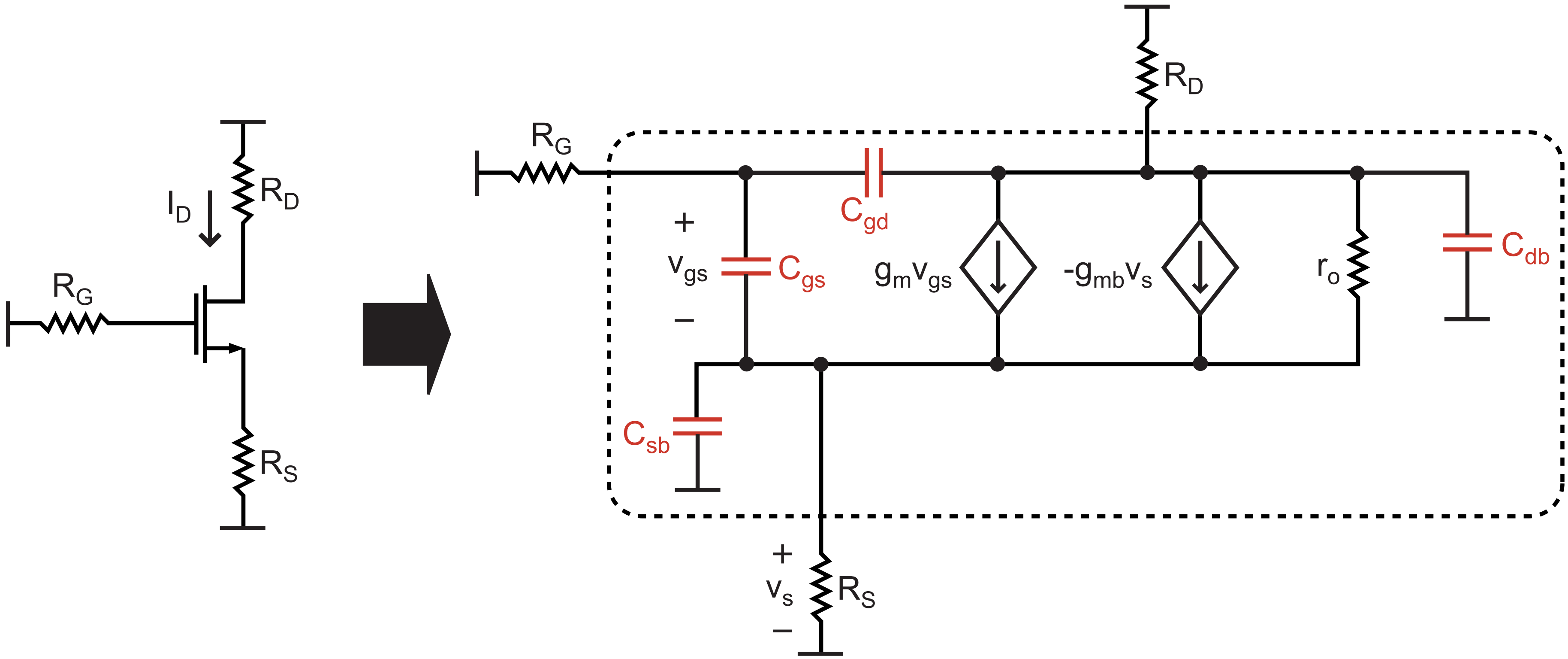
drain to bulk cap: $C_{jsd} = \dfrac{C_j(0)}{\sqrt{1 + V_{DB}/|\Phi_B|}} WE + \dfrac{C_{jsw}(0)}{\sqrt{1 + V_{DB}/|\Phi_B|}} (W + 2E)$

overlap cap: $C_{ov} = WL_D C_{ox} + WC_{fringe}$

gate to channel cap: $C_{gc} = \frac{2}{3} C_{ox} W(L - 2L_D)$

channel to bulk cap: C_{cb} - ignore in this class

CMOS Hybrid-π Model with Caps (Device in Saturation)



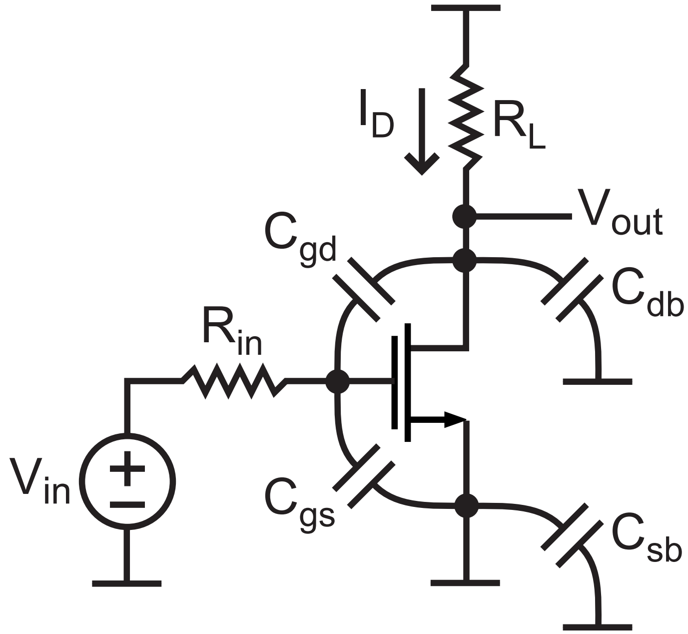
$$C_{gs} = C_{gc} + C_{ov} = \frac{2}{3} C_{ox} W(L-2L_D) + C_{ov}$$

$$C_{gd} = C_{ov}$$

$C_{sb} = C_{jsb}$ (area + perimeter junction capacitance)

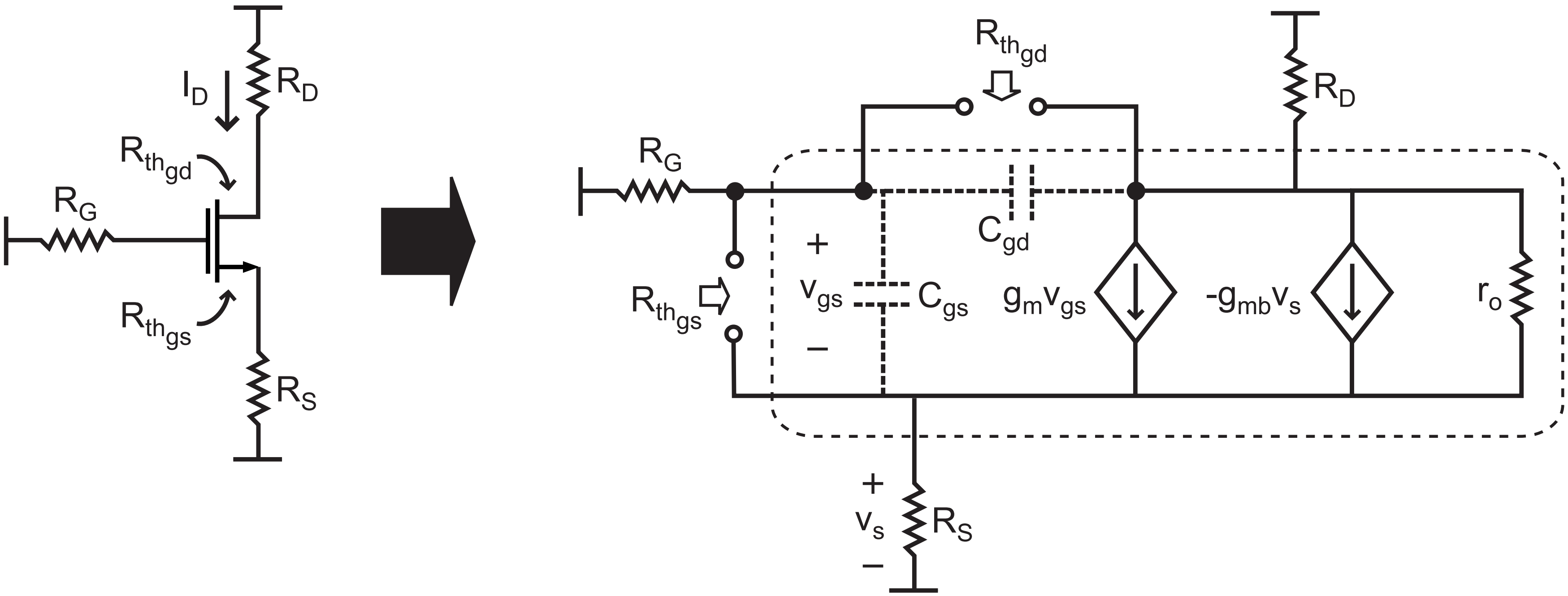
$C_{db} = C_{jdb}$ (area + perimeter junction capacitance)

Back to Common Source Amplifier



- **Of the above capacitors, only C_{gs}, C_{gd}, and C_{db} must be considered**
  - **C_{sb} is grounded on both sides**
- **Thevenin resistance calculations**
  - **C_{db}: R_{thd} || R_d**
  - **C_{gs} and C_{gd}: these involve new Thevenin resistance calculations**

OCT Thevenin Resistance Calculations



- **C_{gs}: Thevenin resistance between gate and source**

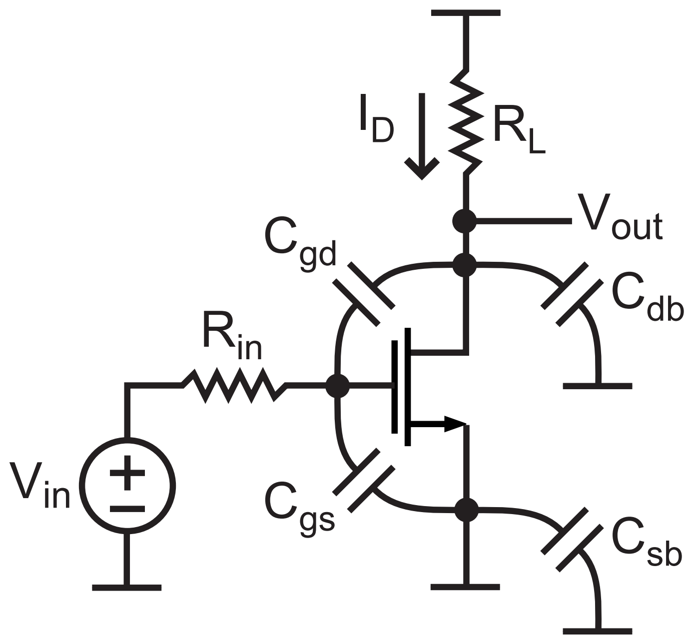
$$R_{th_{gs}} = \frac{R_S(1 + R_D/r_o) + R_G(1 + (g_{mb} + 1/r_o)R_S + R_D/r_o)}{1 + (g_m + g_{mb})R_S + (R_S + R_D)/r_o}$$

- **C_{gd}: Thevenin resistance between gate and drain**

$$R_{th_{gd}} = (R_D + R_G)(1 - r_{ods}/r_o) + r_{ods} g_m R_G$$

$$\text{where } r_{ods} = r_o || \frac{R_D}{1 + (g_m + g_{mb})R_S}$$

OCT Calculations for Common Source Amplifier



- **Estimated bandwidth from OCT method:**

$$BW \approx \frac{1}{\sum_{j=1}^{m} R_{thj} C_j} = \frac{1}{(R_{th_d} || R_d)\, C_{db} + R_{th_{gd}} C_{gd} + R_{th_{gs}} C_{gs}} \; rad/s$$

  - **The above calculations are straightforward given the Thevenin resistance formulas for R_{thd}, R_{thgd}, and R_{thgs}**