Analysis and Design of Analog Integrated Circuits
Lecture 16

Subthreshold Operation and g_m/I_d Design

Michael H. Perrott
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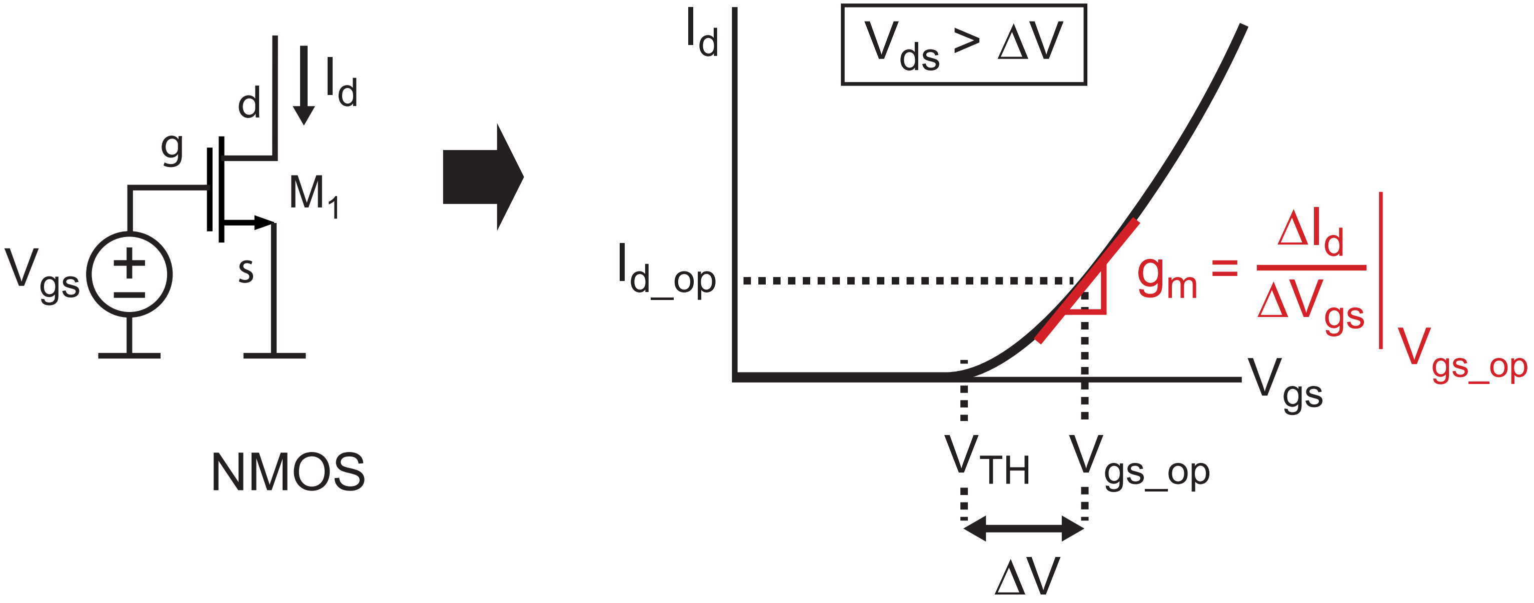
A Closer Look at Transconductance



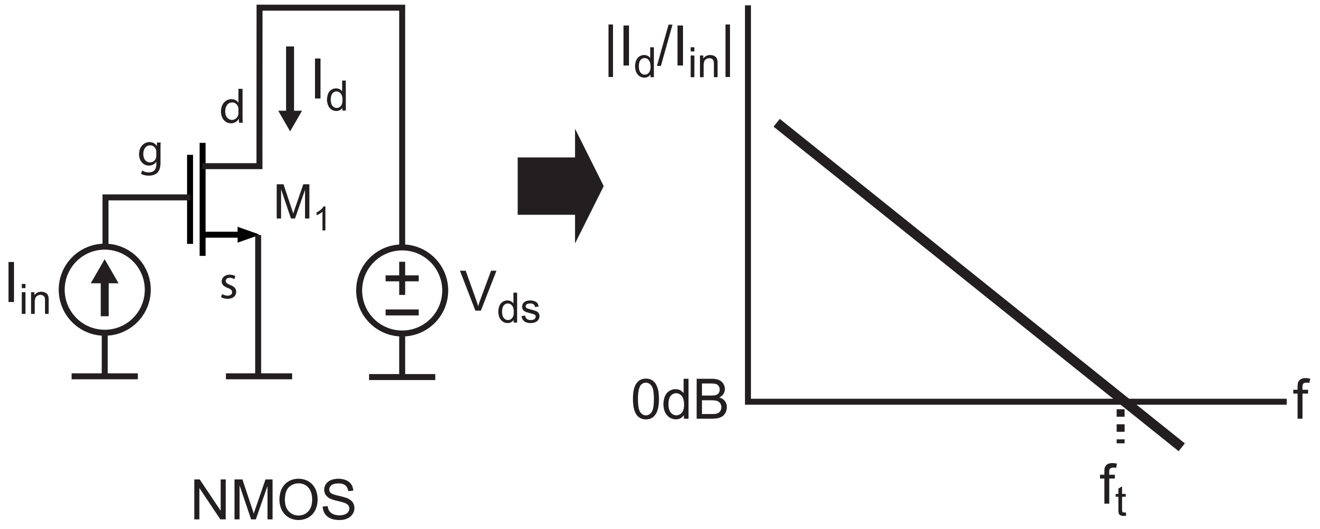
- **Assuming device is in strong inversion and in saturation:**

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{gs} - V_{TH})^2(1 + \lambda V_{ds})$$

$$\Rightarrow \quad g_m = \frac{\delta I_d}{\delta V_{gs}} \approx \mu_n C_{ox}\frac{W}{L}(V_{gs} - V_{TH}) \approx \sqrt{2\mu_n C_{ox}\frac{W}{L}I_d}$$

$$\Rightarrow \quad g_m \approx \frac{I_d\sqrt{2\mu_n C_{ox} W/L}}{\sqrt{I_d}} \approx \boxed{\frac{2I_d}{(V_{gs} - V_{TH})}}$$

Unity Gain Frequency for Current Gain, f_t



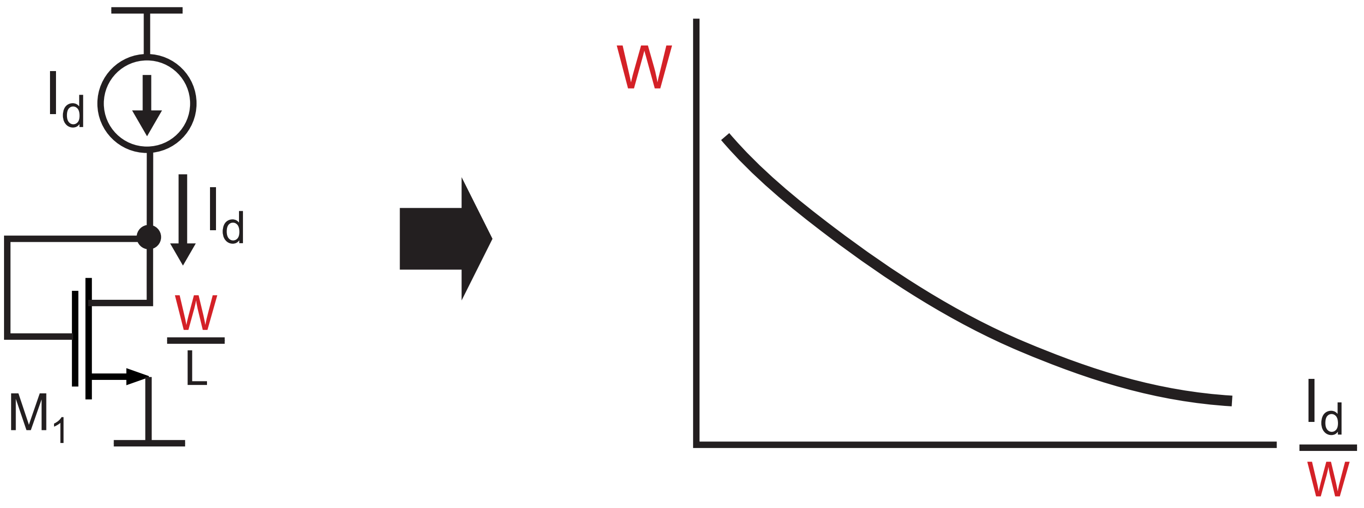
- **Under fairly general conditions, we calculate:**

$$I_d(s) \approx I_{in}(s)\frac{1}{s(C_{gs}+C_{gd})}g_m \quad \Rightarrow \frac{I_d(s)}{I_{in}(s)} \approx \frac{g_m}{s(C_{gs}+C_{gd})}$$

$$\Rightarrow \boxed{f_t = \frac{g_m}{2\pi(C_{gs}+C_{gd})}}$$

- **f_t is a key parameter for characterizing the achievable gain·bandwidth product with circuits that use the device**

Current Density as a Key Parameter



- **Current density is defined as the ratio I_d/W:**
  - **We'll assume that current density is altered by keeping I_d fixed such that only W varies**
    - Maintains constant power
    - r_o (i.e., $1/g_{ds} = 1/(\lambda I_d)$) will remain somewhat constant

Investigating Impact of Current Density

- **For simplicity, let us assume that the CMOS device follows the square law relationship**

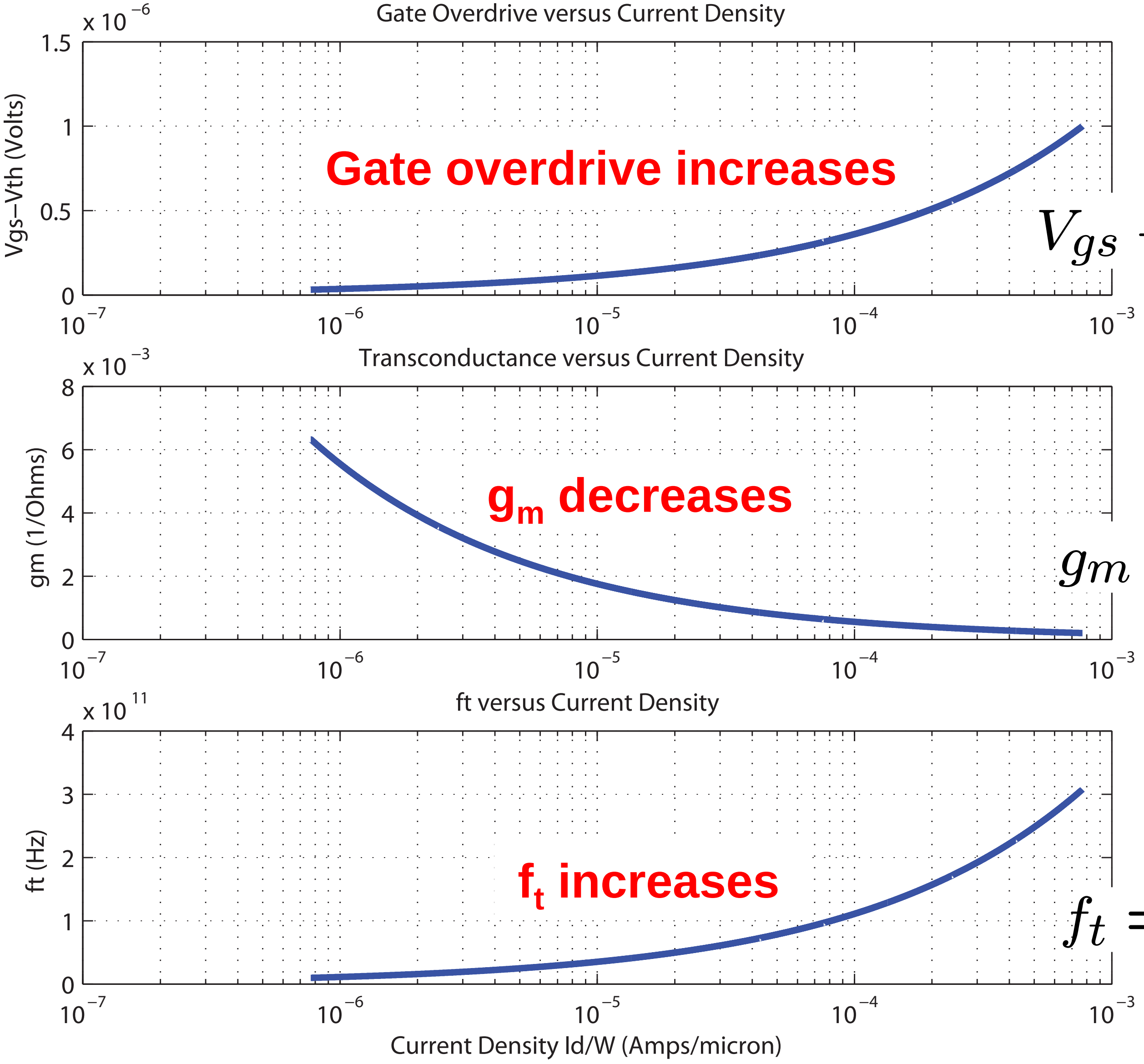
$$I_D \approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{gs} - V_{TH})^2$$

  - **This will lead to the formulations:**

$$V_{gs} - V_{TH} \approx \sqrt{\frac{2L}{\mu_n C_{ox}}\left(\frac{I_d}{W}\right)} \qquad g_m \approx \frac{2I_d}{V_{gs} - V_{TH}}$$

  - **These formulations are only accurate over a narrow region of strong inversion (with the device in saturation)**
  - **However, the general trends observed from the above expressions as a function of current density will provide useful insight**

Investigate the Impact of Increasing Current Density



Gate overdrive increases

$$V_{gs} - V_{TH} \approx \sqrt{\frac{2L}{\mu_n C_{ox}}\left(\frac{I_d}{W}\right)}$$

g_m decreases

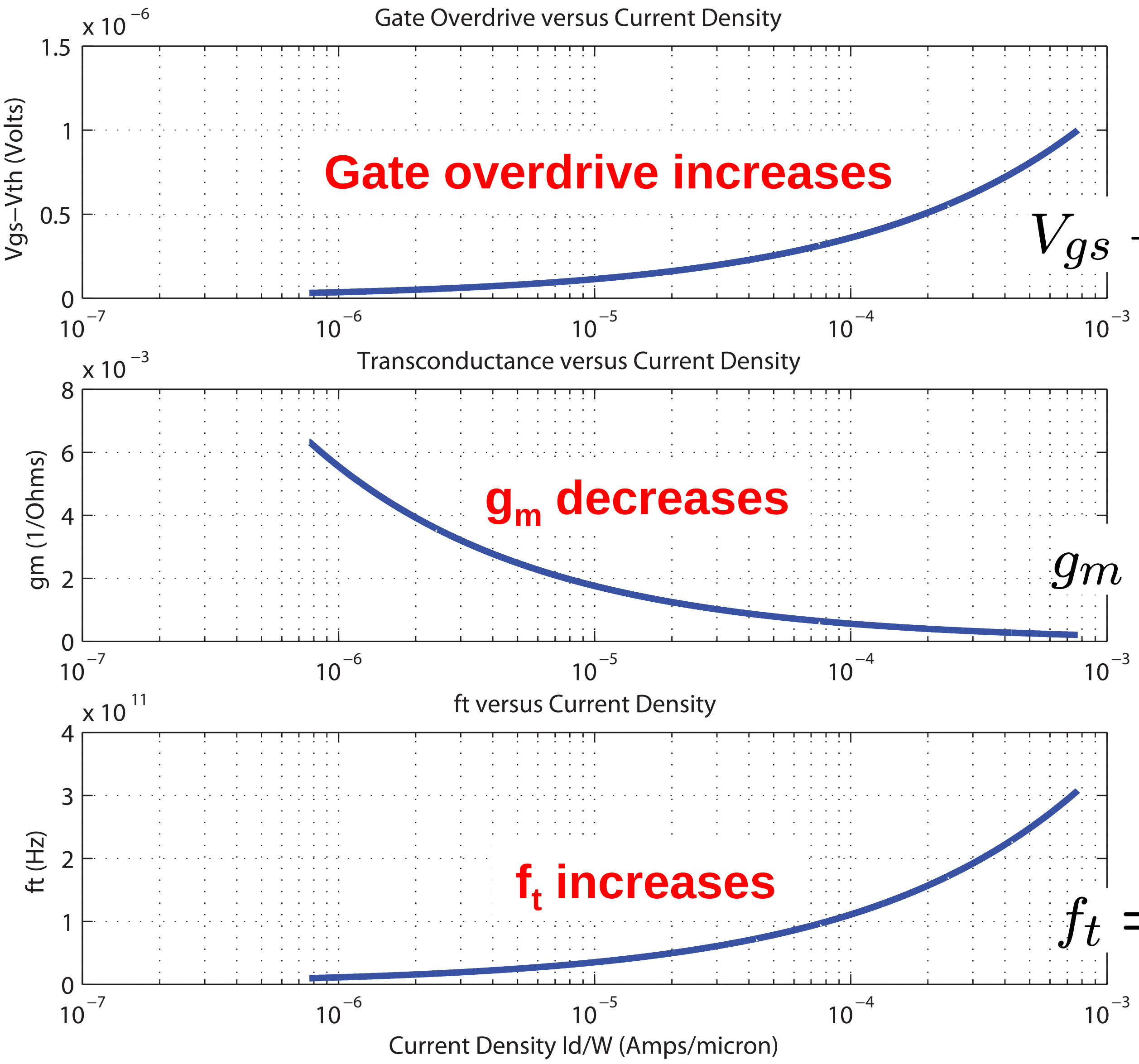
$$g_m \approx \frac{2I_d}{V_{gs} - V_{TH}}$$

f_t increases

$$f_t = \frac{1}{2\pi}\frac{g_m}{C_{gs}} \propto \frac{\sqrt{W}}{W}$$

W decreased with fixed I_d

Transconductance Efficiency Versus f_t



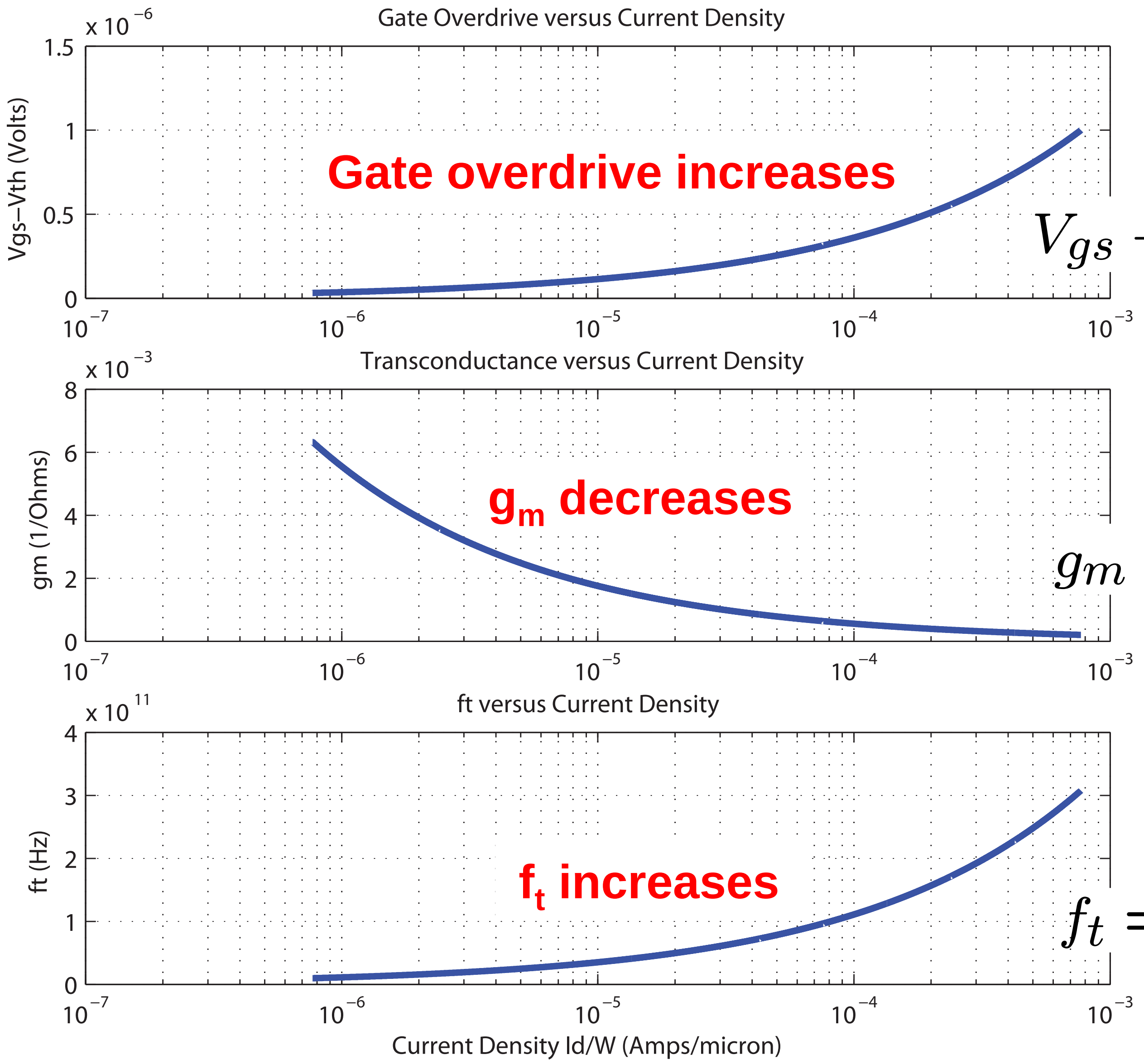
$$V_{gs} - V_{TH} \approx \sqrt{\frac{2L}{\mu_n C_{ox}}\left(\frac{I_d}{W}\right)}$$

$$g_m \approx \frac{2I_d}{V_{gs} - V_{TH}}$$

$$f_t = \frac{1}{2\pi}\frac{g_m}{C_{gs}} \propto \frac{\sqrt{W}}{W}$$

Higher g_m (more gain) ← → Higher f_t (faster speed)

Transistor “Inversion” Operating Regions



Gate overdrive increases

g_m decreases

f_t increases

$$V_{gs} - V_{TH} \approx \sqrt{\frac{2L}{\mu_n C_{ox}}\left(\frac{I_d}{W}\right)}$$

$$g_m \approx \frac{2I_d}{V_{gs} - V_{TH}}$$

$$f_t = \frac{1}{2\pi}\frac{g_m}{C_{gs}} \propto \frac{\sqrt{W}}{W}$$

Weak **Moderate** **Strong**

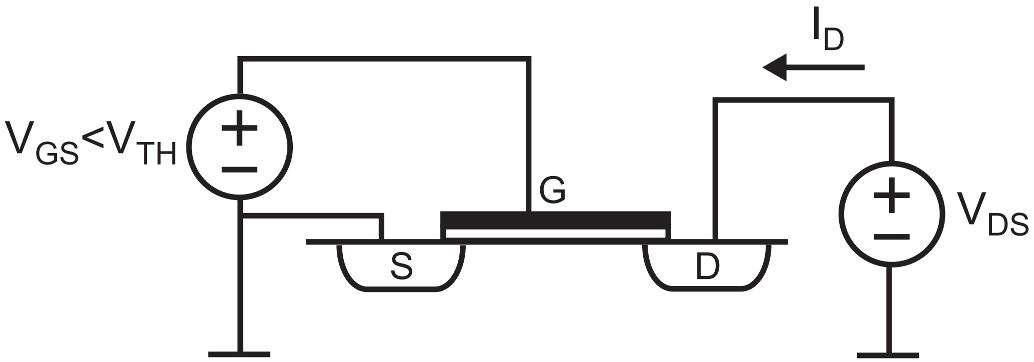
Key Insights Related to Current Density

- **Current density sets the device operating mode**
  - **Weak inversion (subthreshold): highest g_m efficiency**
    - Achieves highest g_m for a given amount of current, I_d
  - **Strong inversion: highest f_t**
    - Achieves highest speed for a given amount of current, I_d
  - **Moderate inversion: compromise between the two**
    - Often the best choice for circuits that do not demand the highest speed but cannot afford the low speed of weak inversion (subthreshold operation)
- **Key issue: validity of square law current assumption**

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{gs} - V_{TH})^2(1 + \lambda V_{ds})$$

  - **The above is only accurate over a narrow range of strong inversion (i.e., the previous plots are inaccurate)**
    - General observations above are still true, though

A Proper Model for Subthreshold Operation



- **Drain current:**

$$I_D = I_{D0}\frac{W}{L}e^{V_{gs}/(nV_t)}\left(1 - e^{-V_{ds}/V_t}\right)$$

  - **Where:**

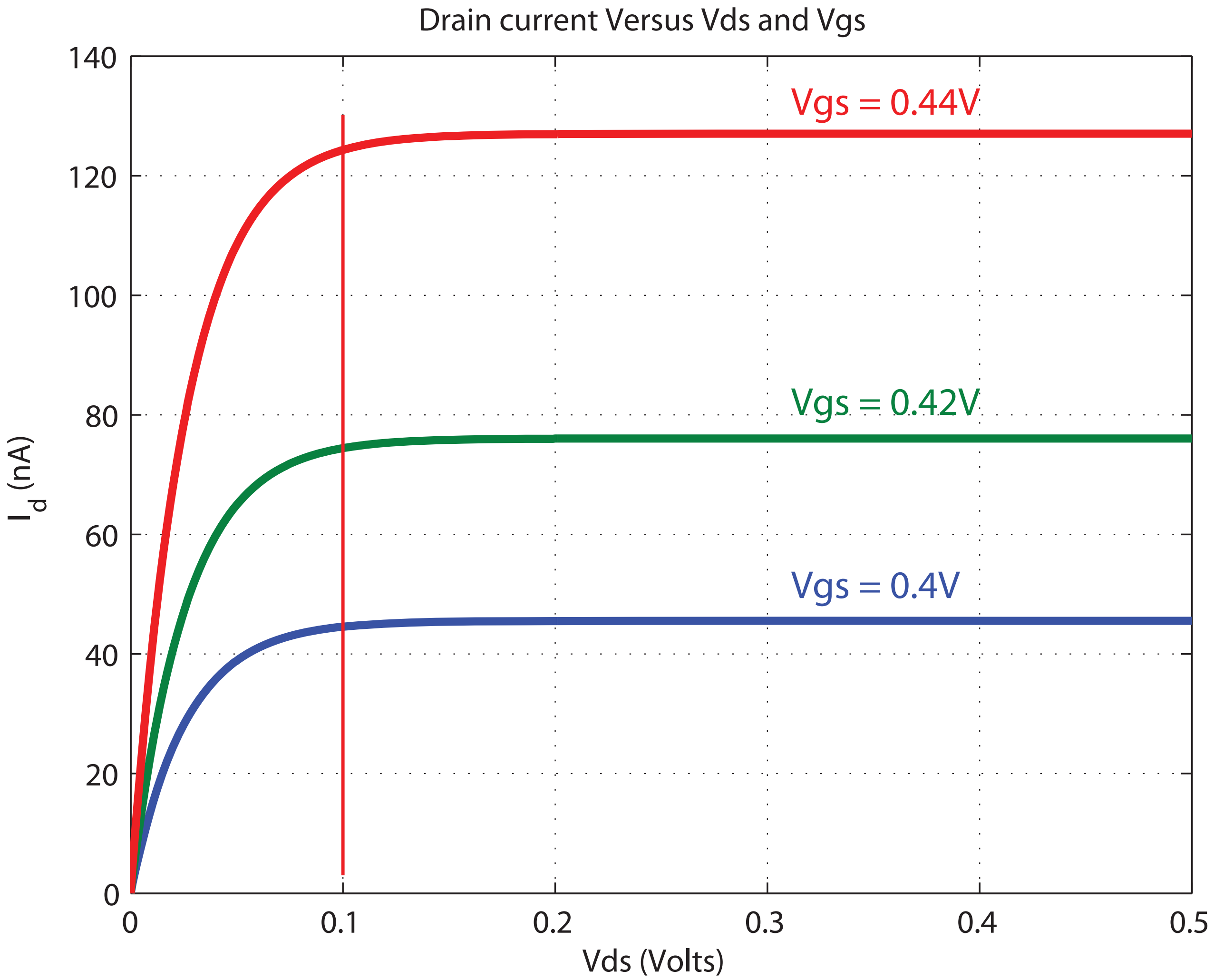
$$V_t = \frac{kT}{q} \approx 26\text{mV} \;\; \text{at} \;\; T = 300K$$

$$n = \frac{C_{ox} + C_{depl}}{C_{ox}} \approx 1.5$$

$$I_{D0} = \mu_n C_{ox}(n-1)V_t^2 e^{-V_{TH}/(nV_t)}$$

  - **Note: channel length modulation, i.e., λ, is ignored here**

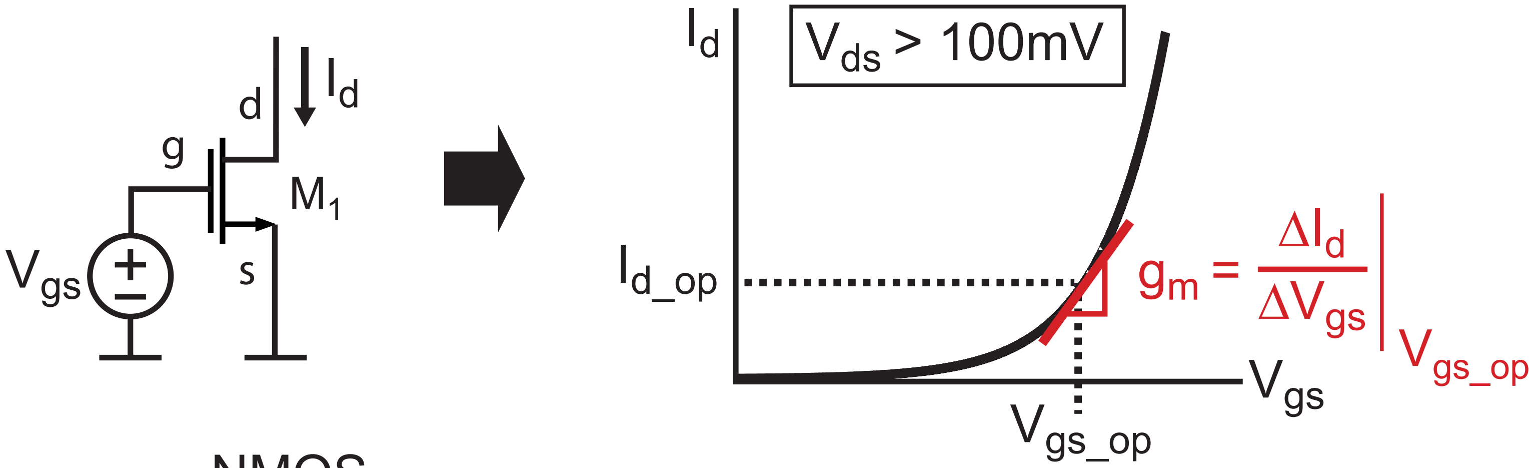
Saturation Region for Subthreshold Operation



- **Saturation occurs at roughly V_{ds} > 100 mV**

$$\Rightarrow \; I_D = I_{D0}\frac{W}{L}e^{V_{gs}/(nV_t)}\left(1 - e^{-V_{ds}/V_t}\right) \approx I_{D0}\frac{W}{L}e^{V_{gs}/(nV_t)}$$

Transconductance in Subthreshold Region



NMOS

- **Assuming device is in subthreshold and in saturation:**

$$I_D \approx I_{D0}\frac{W}{L}e^{V_{gs}/(nV_t)}$$

$$\Rightarrow \quad g_m = \frac{\delta I_d}{\delta V_{gs}} \approx I_{D0}\frac{W}{L}e^{V_{gs}/(nV_t)}\frac{1}{nV_t} = \boxed{\frac{I_d}{nV_t}}$$

g_m purely a function of I_d!

Recall for strong inversion : $g_m \approx \boxed{\frac{2I_d}{(V_{gs} - V_{TH})}}$

Comparison of Strong and Weak Inversion for g_m

- Assumption: I_d is constant with only W varying
- Strong inversion formulation predicts ever increasing g_m with reduced overdrive voltage

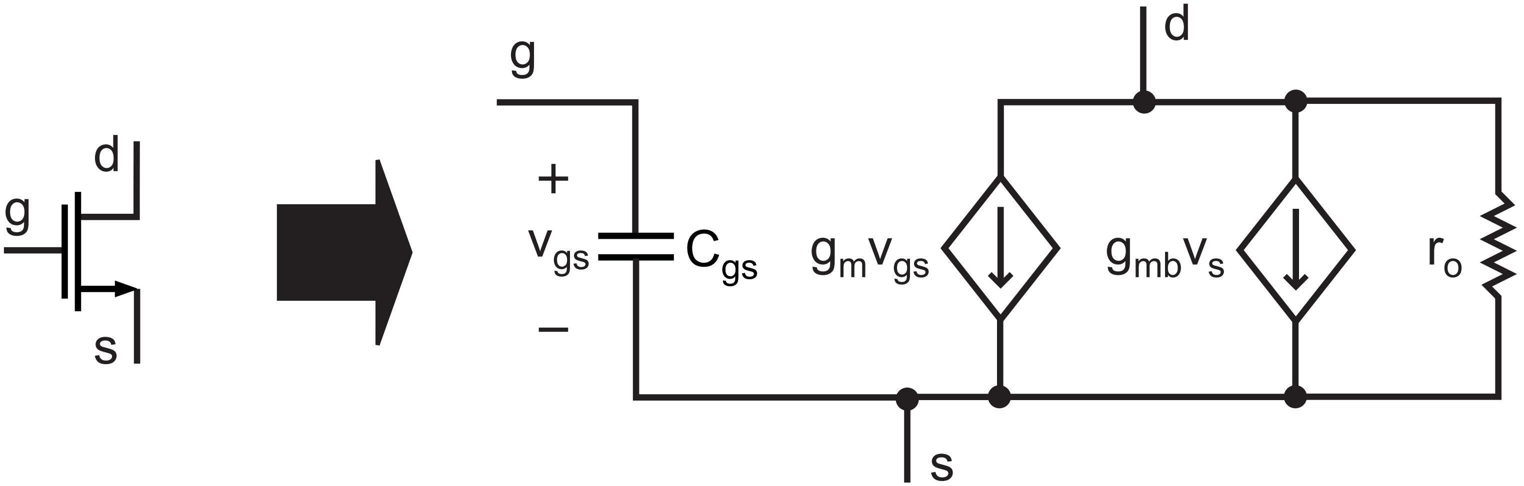
$$g_m \approx \frac{2I_d}{(V_{gs} - V_{TH})}$$

  - Reduced current density leads to reduced overdrive voltage and therefore higher g_m
- Weak inversion formulation predicts that g_m will hit a maximum value as current density is reduced

$$g_m =\approx \frac{I_d}{nV_t}$$

  - Note that the area of the device no longer influences g_m when operating in weak inversion (i.e., subthreshold)

Hybrid-π Model in Subthreshold Region (In Saturation)

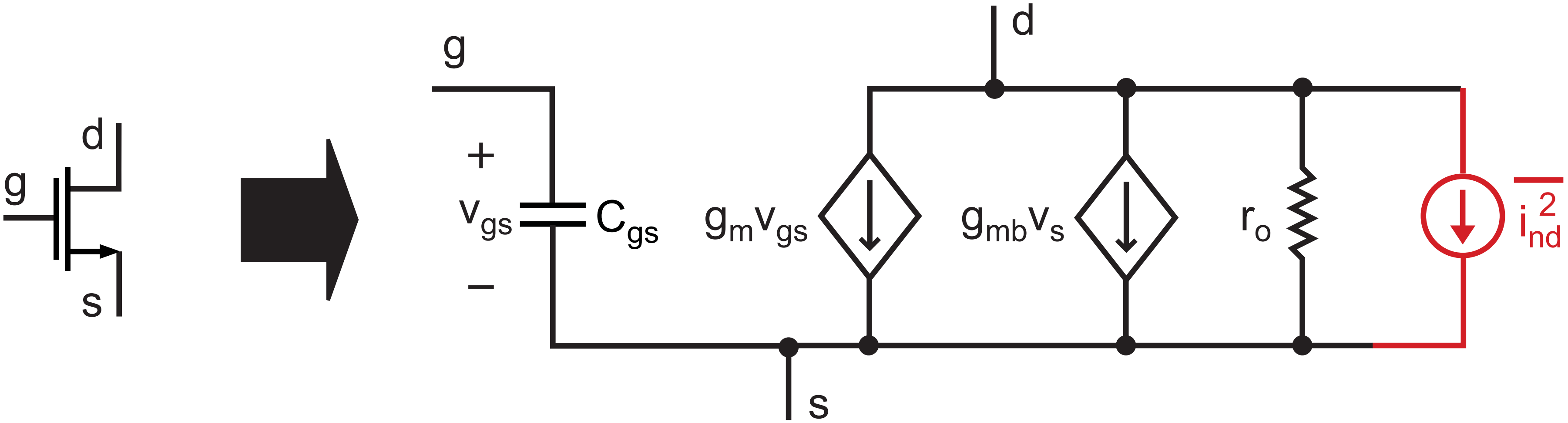


- **Looks the same in form as for strong inversion, but different expressions for the various parameters**

$$g_m \approx \left(\frac{1}{n}\right)\frac{I_d}{V_t} \qquad g_{mb} \approx \left(\frac{n-1}{n}\right)\frac{I_d}{V_t} \qquad r_o \approx \frac{1}{\lambda I_d}$$

  - **We can use the very same Thevenin modeling approach as in strong inversion**
    - We just need to calculate g_m and g_{mb} differently

Noise for Subthreshold Operation (In Saturation)



- **Recall transistor drain noise in strong inversion:**

$$\overline{i_{nd}^2} = \underbrace{4kT\gamma g_{dso}\Delta f}_{\textbf{Thermal noise}} + \underbrace{\frac{K_f}{f}\frac{g_m^2}{WLC_{ox}^2}\Delta f}_{\textbf{1/f noise}}$$

- **In weak inversion (i.e., subthreshold):**

$$\overline{i_{nd}^2} = \underbrace{2kTng_m\Delta f}_{\textbf{Thermal noise}} + \underbrace{\frac{K_f}{f}\frac{g_m^2}{WLC_{ox}^2}\Delta f}_{\textbf{1/f noise}}$$

Strong Inversion Versus Weak Inversion

- **Strong inversion ($V_{gs} > V_{TH}$)**
  - **Poor g_m efficiency (i.e., g_m/I_d is low) but fast speed**
  - **Need $V_{ds} > (V_{gs} - V_{TH}) = \Delta V$ to be in saturation**
  - **Key device parameters are calculated as:**

$$g_m \approx \frac{2I_d}{(V_{gs} - V_{TH})} \qquad g_{mb} \approx \frac{\gamma g_m}{2\sqrt{2|\Phi_F| + V_{SB}}} \qquad r_o \approx \frac{1}{\lambda I_d}$$

- **Weak inversion ($V_{gs} < V_{TH}$)**
  - **Good g_m efficiency (i.e., g_m/I_d is high) but slow speed**
  - **Need V_{ds} > 100mV to be in saturation**
  - **Key device parameters are calculated as:**

$$g_m \approx \left(\frac{1}{n}\right)\frac{I_d}{V_t} \qquad g_{mb} \approx \left(\frac{n-1}{n}\right)\frac{I_d}{V_t} \qquad r_o \approx \frac{1}{\lambda I_d}$$

- **Moderate inversion: compromise between the two**

Thevenin Modeling Techniques Can Be Applied to All Cases

g_m/I_d Design

- **g_m/I_d design is completely SPICE based**
  - **Hand calculations of g_m, r_o, etc. are not performed**
- **Various transistor parameters are plotted in terms of g_m/I_d**
  - **Low g_m/I_d corresponds to strong inversion**
  - **High g_m/I_d corresponds to weak inversion**
- **Once a given value of g_m/I_d is chosen, it constrains the relationship between W, L, f_t, etc. such that the sizing of devices becomes a straightforward exercise**

Useful References Related to g_m/I_d Design

- **Prof. Bernhard Boser's Lecture:**
  - B. E. Boser, "Analog Circuit Design with Submicron Transistors," IEEE SSCS Meeting, Santa Clara Valley, May 19, 2005, http://www.ewh.ieee.org/r6/scv/ssc/May1905.htm
- **Prof. Boris Murmann's Course Notes:**
  - **https://ccnet.stanford.edu/cgi-bin/course.cgi?cc=ee214&action=handout_view&V_section=general**
    - See Slides 45 to 67 in particular
- **Prof. Reid Harrison's paper on a low noise instrument amplifier:**
  - **http://www.ece.utah.edu/~harrison/JSSC_Jun_03.pdf**

Summary

- **CMOS devices in saturation can be utilized in weak, moderate, or strong inversion**
  - **Each region of operation involves different expressions for drain current as a function of V_{gs} and V_{ds}**
  - **It is best to use SPICE to calculate parameters such as g_m, g_{mb}, r_o due to the complexity of the device model in encompassing these three operating regions**
    - g_m/I_d methodology is one such approach
  - **Weak inversion offers large g_m/I_d but slow speed, and strong inversion offers fast speed but lower g_m/I_d**
  - **Moderate inversion offers the best compromise between achieving reasonable g_m/I_d and reasonable speed**
- **Thevenin modeling approach is valid for all operating regions once g_m, g_{mb}, and r_o are known**