Analysis and Design of Analog Integrated Circuits
Lecture 15

Mismatch and Nonlinearity

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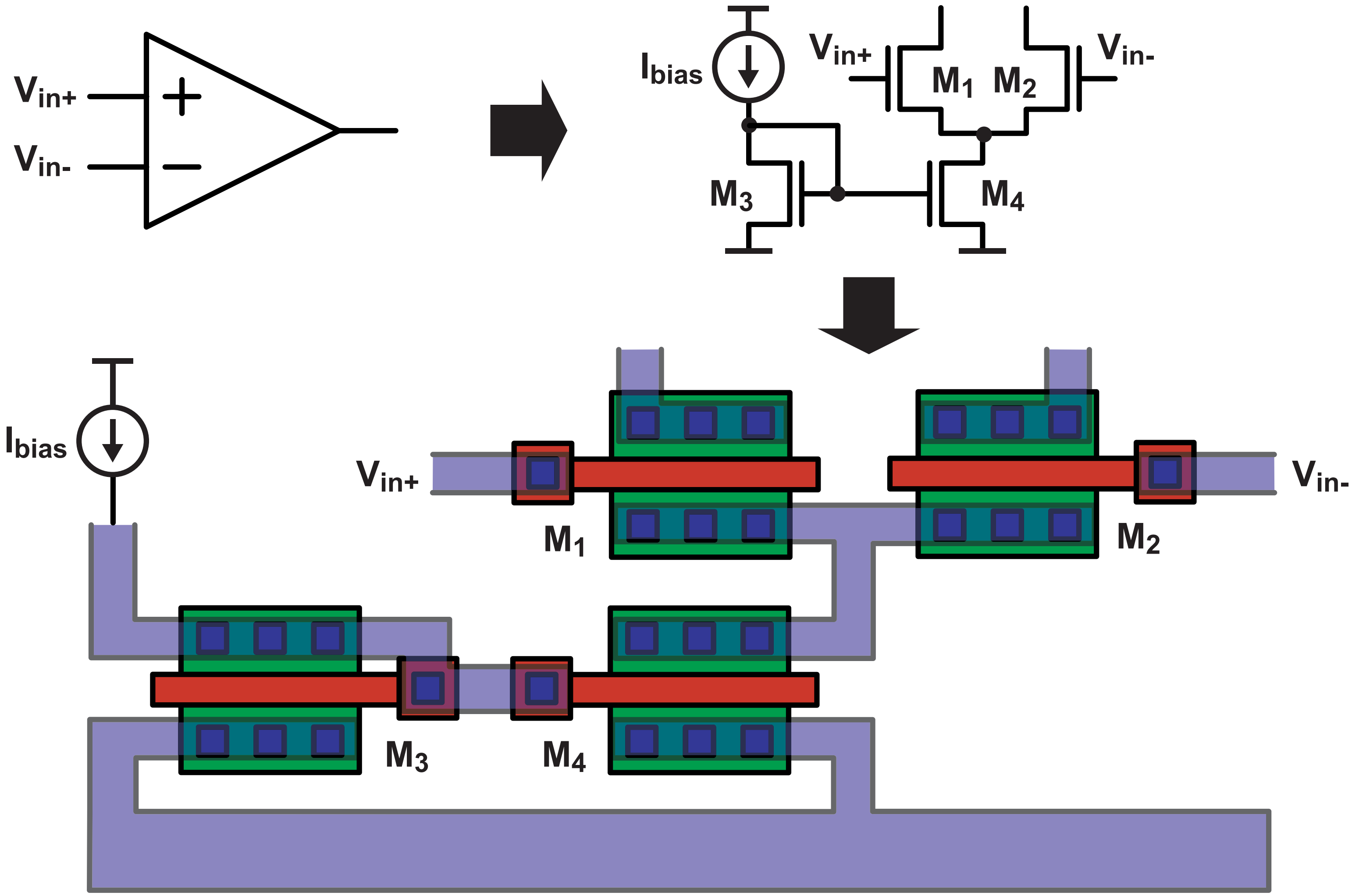
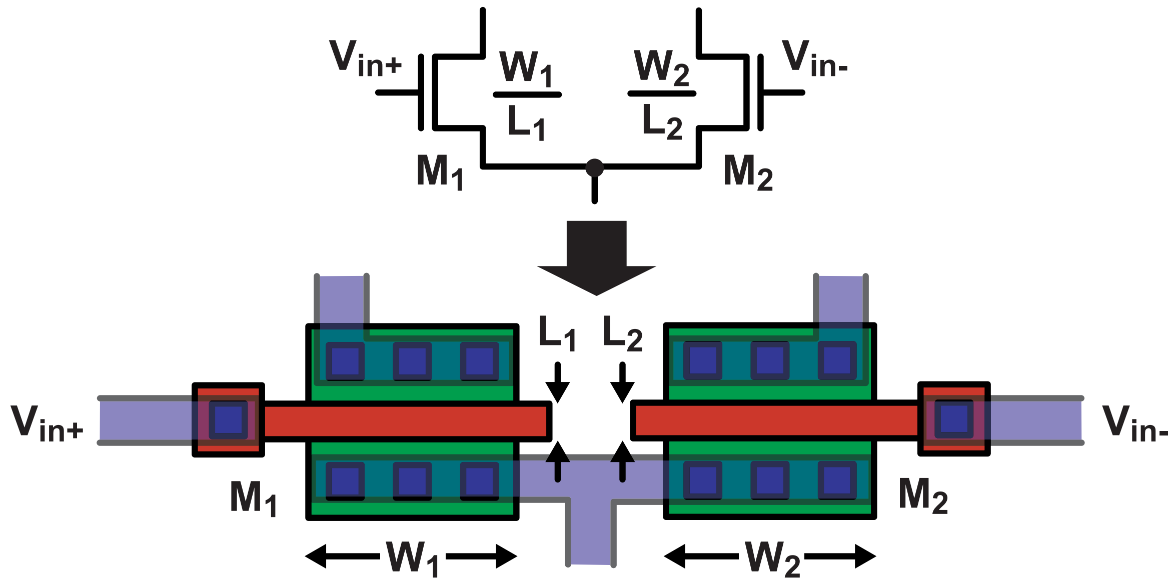
A Closer Look at Differential Pairs



- **Fabrication of devices comes with variation**
  - **Width, length, and $\mu_n C_{ox}$ mismatch between devices**
  - **Threshold voltage mismatch between devices**

Modeling the Impact of Mismatch in MOS Devices



- **Compare the drain current of devices in saturation:**
  - **Assume M_1 has current:**

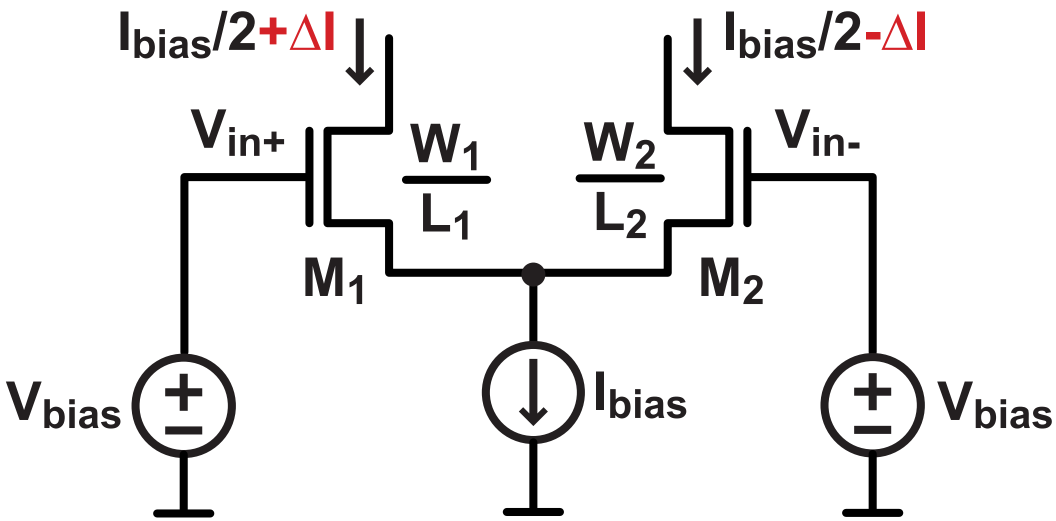
$$I_{D1} \approx \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{gs} - V_{TH})^2$$

  - **Assume that M_2 is mismatched to M_1:**

$$I_{D2} \approx \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L} + \Delta \frac{W}{L} \right) (V_{gs} - (V_{TH} + \Delta V_{TH}))^2$$

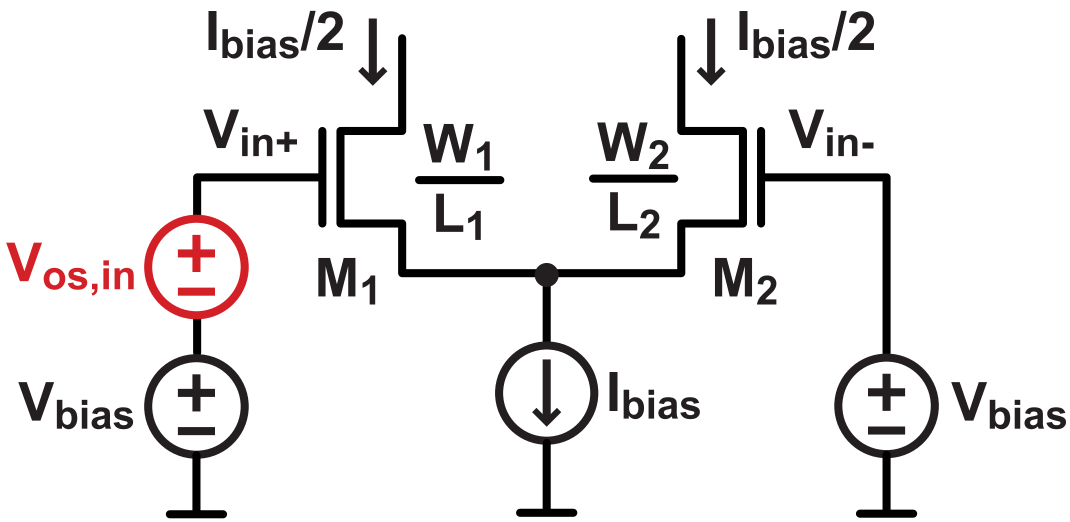
    - Note than $\mu_n C_{ox}$ mismatch is lumped into Δ(W/L)

Key Impact of Mismatch



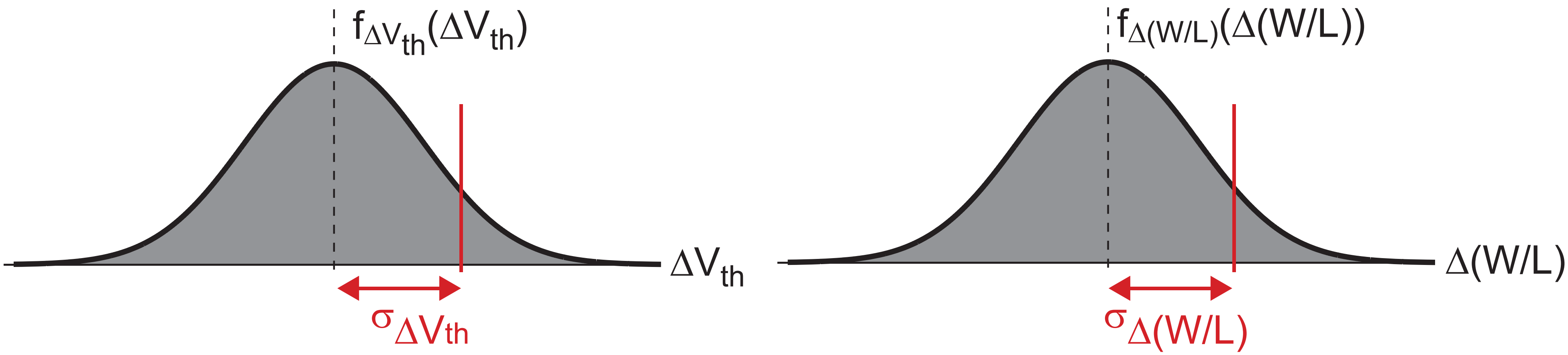
- Ideally, a differential pair will yield identical output currents assuming identical input voltages for V_{in+} and V_{in-}
- In the case of mismatch, the output currents will *NOT* be equal with equal input voltages

Mismatch-Induced Offset Voltage



- **Define input offset voltage of the differential pair as the input voltage difference required to achieve identical output currents from the differential pair**
  - **Higher mismatch leads to higher offset voltage**

Mismatch Modeled as Random Variables

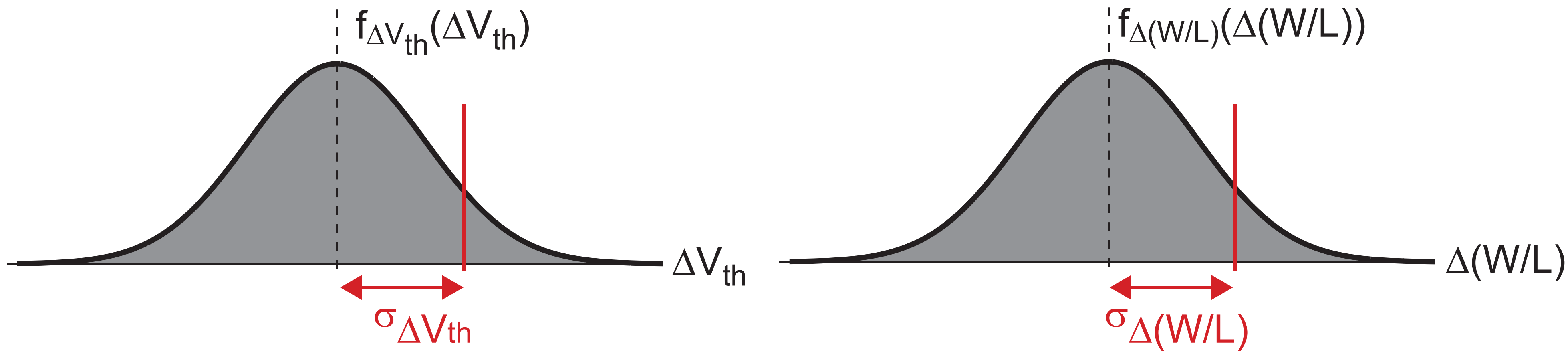


- **We often assume a Gaussian PDF for the *random* portion of mismatch**
  - **The standard deviation of the PDF is the key metric that we often use to approximate the impact of mismatch**

$$\Delta V_{TH} \approx \sigma_{\Delta V_{TH}} \qquad \Delta\frac{W}{L} \approx \sigma_{\Delta\frac{W}{L}}$$

- **Note that there is also a deterministic portion of mismatch called *systematic* mismatch**
  - **Systematic mismatch can often be avoided with proper design and layout techniques**

Estimating Mismatch Parameters



- **Mathematical and experimental investigation has revealed**

$$\sigma_{\Delta V_{TH}} \approx \frac{A_{V_{TH}}}{\sqrt{WL}} \qquad \sigma_{\Delta \frac{W}{L}} \approx \frac{A_K}{\sqrt{WL}}$$

  - ***A_{VTH}* and *A_K* are proportionality factors that are sometimes provided by fabrication reports and sometimes embedded within "Monte-Carlo" device models**

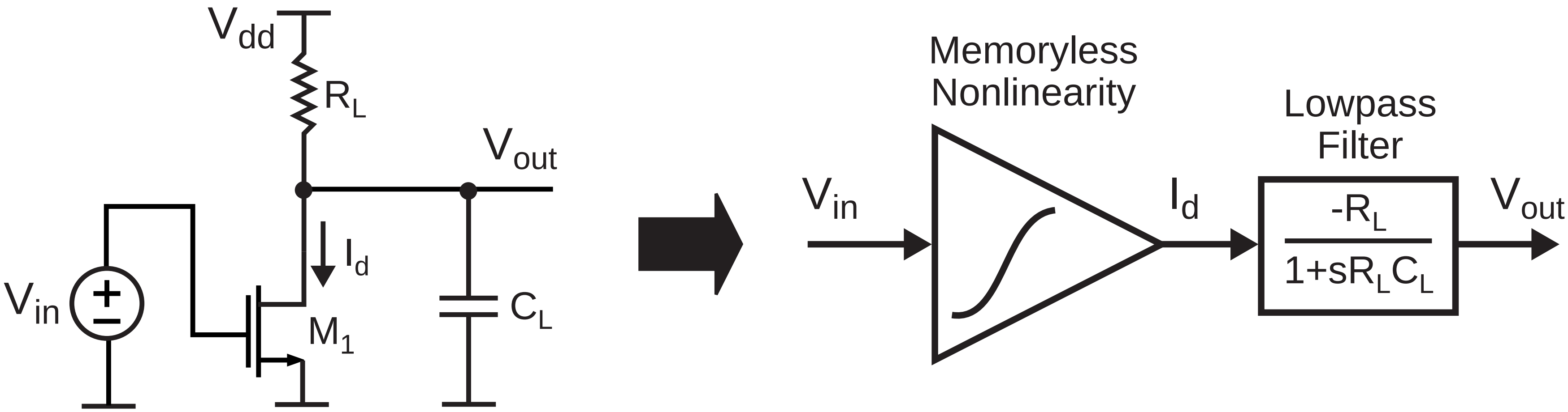
Key insight: better matching achieved with larger devices

More Information on Mismatch

- **Marcel Pelgrom at NXP (formerly Philips) wrote the seminal papers on this topic**
  - **M.J.M. Pelgrom, A.C.J. Duinmaiger, A.P.G. Welbers, "Matching Properties of MOS Transistors," IEEE J. Solid-State Circuits, vol. SC-24, pp. 1433-1439, Oct. 1989**
  - **M.J.M. Pelgrom, H.P. Tuinhout, M. Vertregt, "Transistor Matching in Analog CMOS Applications," IEDM Dig. of Tech. Papers, pp. 34.1.1-34.1.4, Dec. 1998**

Nonlinearities in Amplifiers

- **We can generally break up an amplifier into the cascade of a memoryless nonlinearity and an input and/or output transfer function**

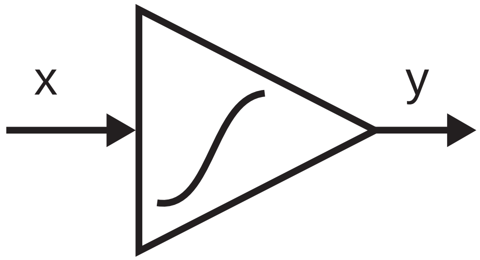


- **Impact of nonlinearities with sine wave input**
  - **Causes harmonic distortion (i.e., creation of harmonics)**
- **Impact of nonlinearities with several sine wave inputs**
  - **Causes harmonic distortion for each input AND intermodulation products**

Impact of nonlinearity often assessed based on issues related to communication system design

Analysis of Amplifier Nonlinearities

- **Focus on memoryless nonlinearity block**
  - **The impact of filtering can be added later**



- **Model nonlinearity as a Taylor series expansion up to its third order term (assumes small signal variation)**

$$y(t) \approx c_o + c_1 x(t) + c_2 x(t)^2 + c_3 x(t)^3$$

  - **For harmonic distortion, consider**

$$x(t) = A\cos(wt)$$

  - **For intermodulation, consider**

$$x(t) = A(\cos(w_1 t) + \cos(w_2 t))$$

Harmonic Distortion

$$y(t) = c_o + c_1 x(t) + c_2 x(t)^2 + c_3 x(t)^3$$

$$\text{where } x(t) = A\cos wt$$

- **Substitute x(t) into polynomial expression**

$$y(t) - c_o = c_1 A\cos wt + c_2 A^2 \cos^2 wt + c_3 A^3 \cos^3 wt$$

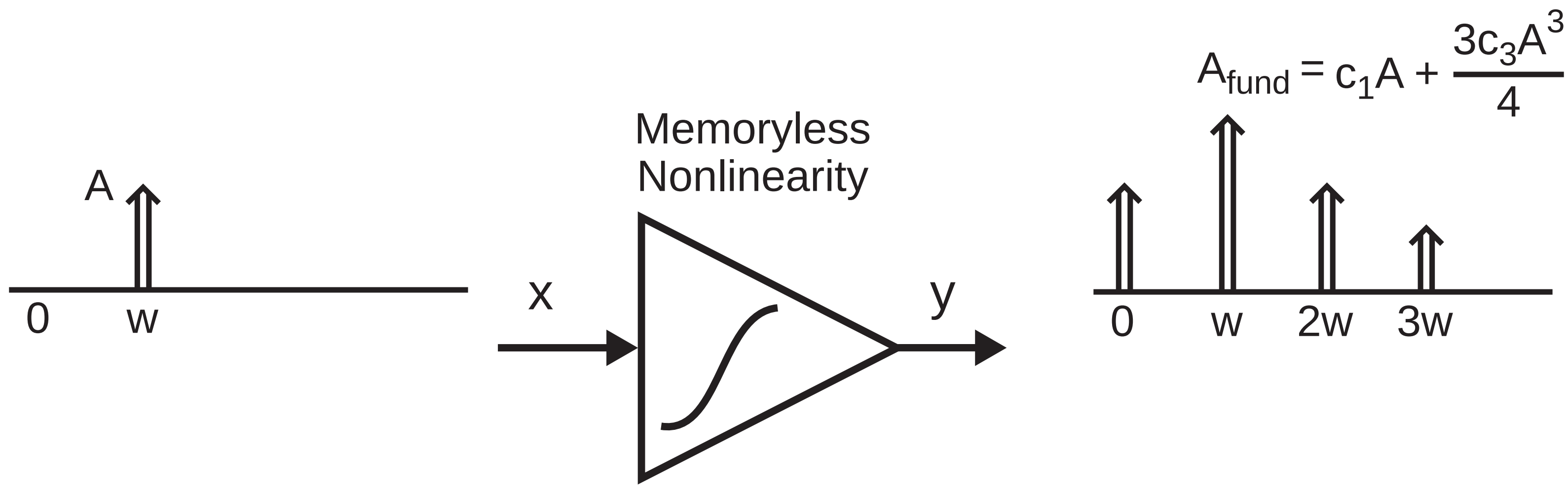
$$= c_1 A\cos wt + \frac{c_2 A^2}{2}(1 + \cos 2wt) + \frac{c_3 A^3}{4}(3\cos wt + \cos 3wt)$$

$$= \frac{c_2 A^2}{2} + \underbrace{\left(c_1 A + \frac{3c_3 A^3}{4}\right)\cos wt}_{\text{Fundamental}} + \underbrace{\frac{c_2 A^2}{2}\cos 2wt + \frac{c_3 A^3}{4}\cos 3wt}_{\text{Harmonics}}$$

Fundamental **Harmonics**

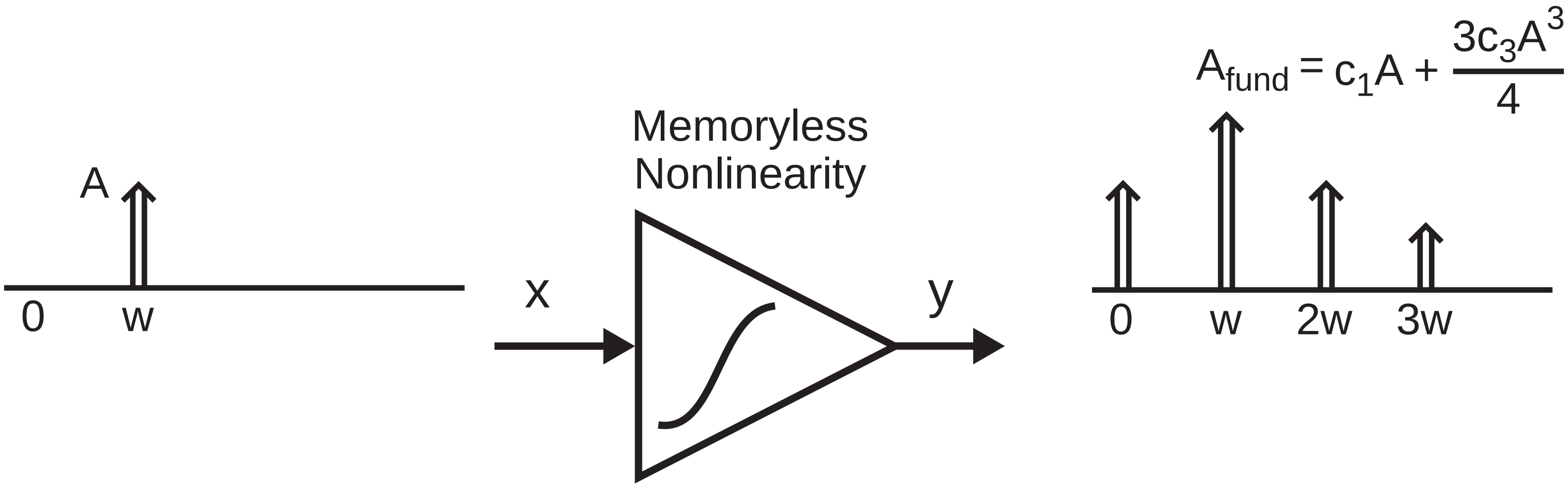
- **Notice that each harmonic term, cos(*nwt*), has an amplitude that grows in proportion to A^n**
  - **Very small for small A, very large for large A**

Frequency Domain View of Harmonic Distortion

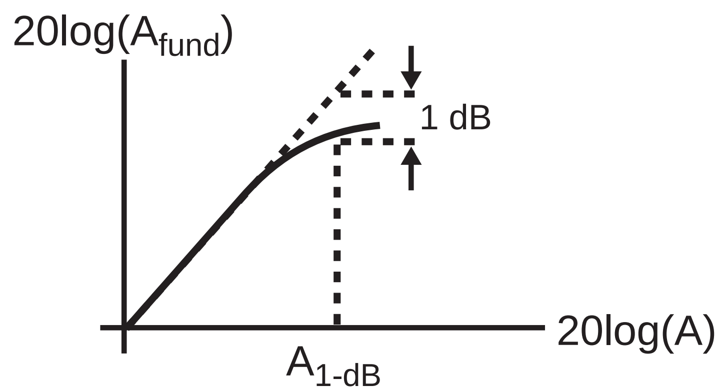


- **Harmonics cause "noise"**
  - **Their impact depends highly on application**
    - Low noise amplifiers (LNA) for wireless systems – typically not of consequence
    - Power amplifiers for wireless systems – can degrade spectral mask
    - Audio amp – depends on your listening preference!
- **Gain for fundamental component depends on input amplitude!**

1 dB Compression Point



- **Definition: input signal level such that the small-signal gain drops by 1 dB**
  - **Input signal level is high!**



- **Typically calculated from simulation or measurement rather than analytically**
  - **Analytical model must include many more terms in Taylor series to be accurate in this context**

Harmonic Products with An Input of Two Sine Waves

$$y(t) = c_o + c_1 x(t) + c_2 x(t)^2 + c_3 x(t)^3$$

$$\text{where } x(t) = A(\cos w_1 t + \cos w_2 t)$$

- **DC and fundamental components**

$$\left(c_o + c_2 A^2\right) + \left(\left(c_1 A + \frac{9}{4} c_3 A^3\right)(\cos w_1 t + \cos w_2 t)\right)$$

- **Second and third harmonic terms**

$$\left(\frac{c_2 A^2}{2}(\cos 2w_1 t + \cos 2w_2 t)\right) + \left(\frac{c_3 A^3}{4}(\cos 3w_1 t + \cos 3w_2 t)\right)$$

- **Similar result as having an input with one sine wave**
  - **But, we haven't yet considered cross terms!**

Intermodulation Products

$$y(t) = c_o + c_1 x(t) + c_2 x(t)^2 + c_3 x(t)^3$$

$$\text{where } x(t) = A(\cos w_1 t + \cos w_2 t)$$

- **Second-order intermodulation (IM2) products**

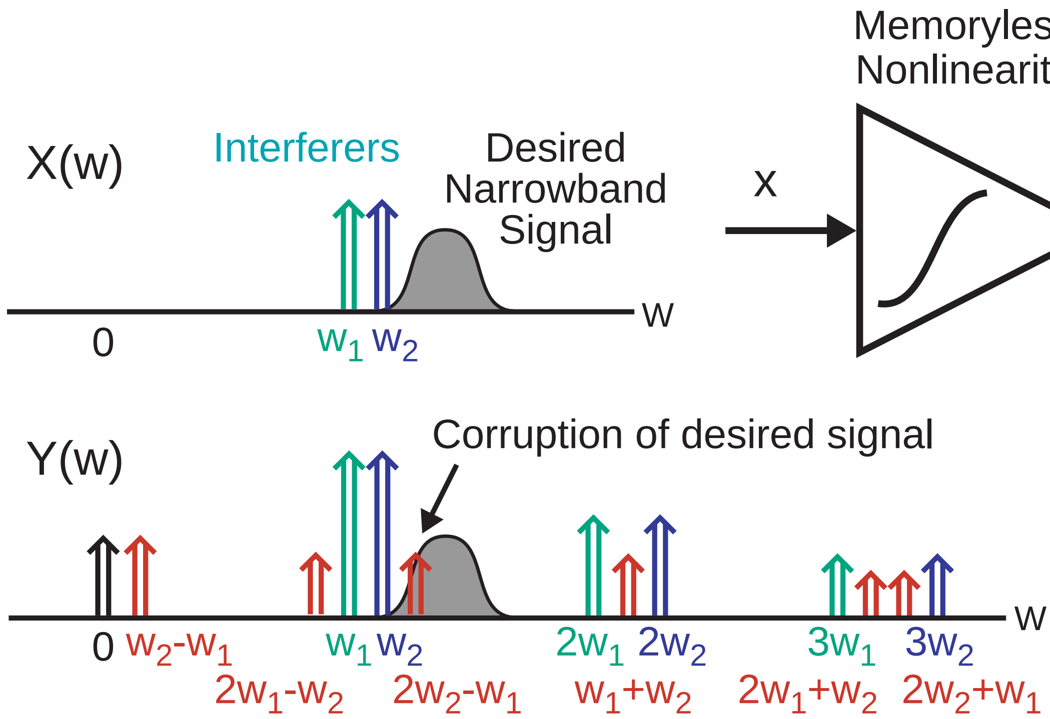
$$c_2 A^2 (\cos(w_1 + w_2)t + \cos(w_2 - w_1)t)$$

- **Third-order intermodulation (IM3) products**

$$\frac{3}{4} c_3 A^3 \Big(\cos(2w_1 + w_2)t + \cos(2w_1 - w_2)t + \cos(2w_2 + w_1) + \cos(2w_2 - w_1)t \Big)$$

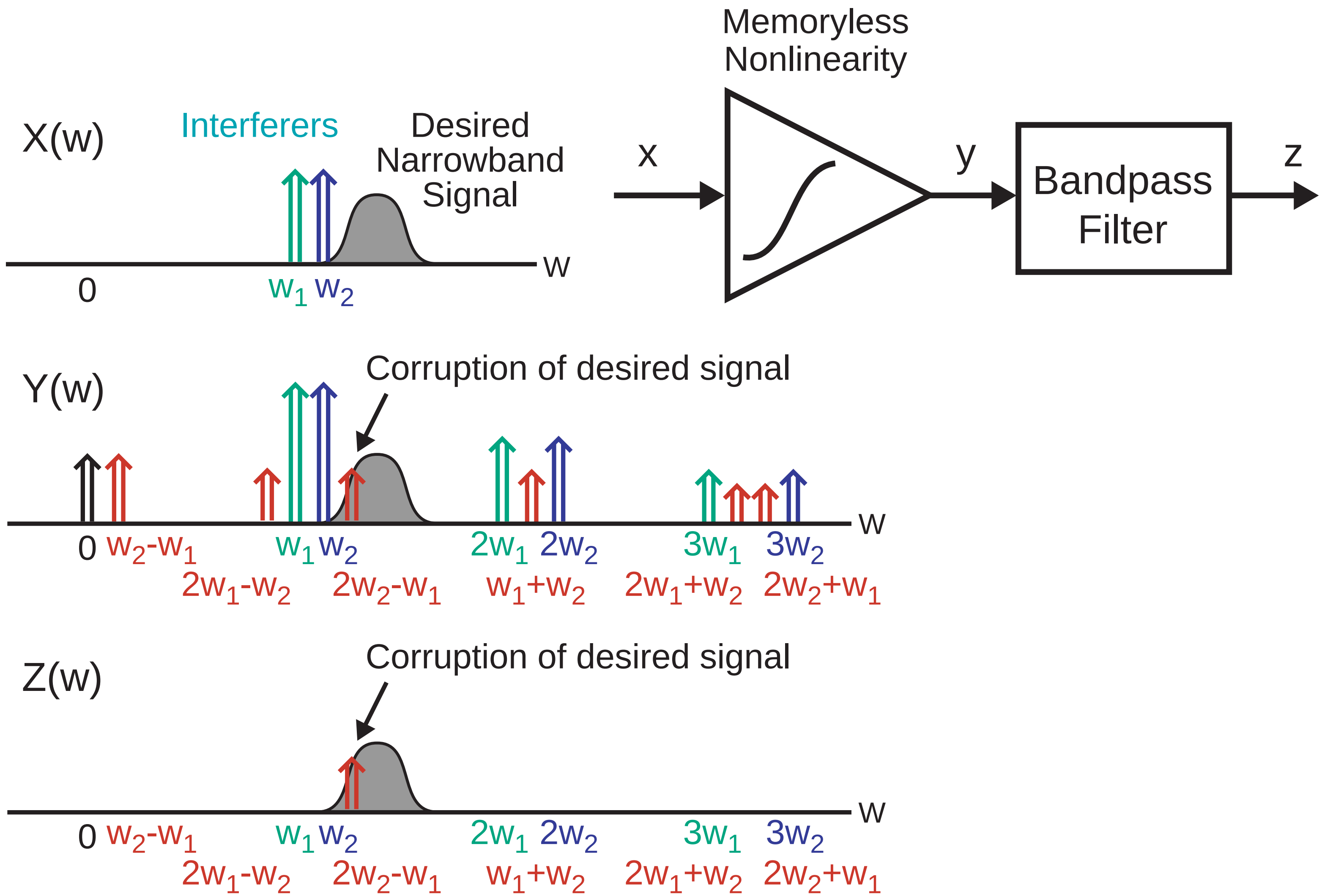
  - **These are the troublesome ones for narrowband wireless systems**

Corruption of Narrowband Signals by Interferers



- **Wireless receivers must select a desired signal that is accompanied by interferers that are often much larger**
  - **LNA nonlinearity causes the creation of harmonic and intermodulation products**
  - **Must remove interference and its products to retrieve desired signal**

Use Filtering to Remove Undesired Interference



- **Ineffective for IM3 term that falls in the desired signal frequency band**

Characterization of Intermodulation

- **Magnitude of third order products is set by c_3 and input signal amplitude (for small A)**

$$\frac{3}{4}c_3A^3\Big(\cos(2w_1+w_2)t+\cos(2w_1-w_2)t + \cos(2w_2+w_1)+\cos(2w_2-w_1)t\Big)$$

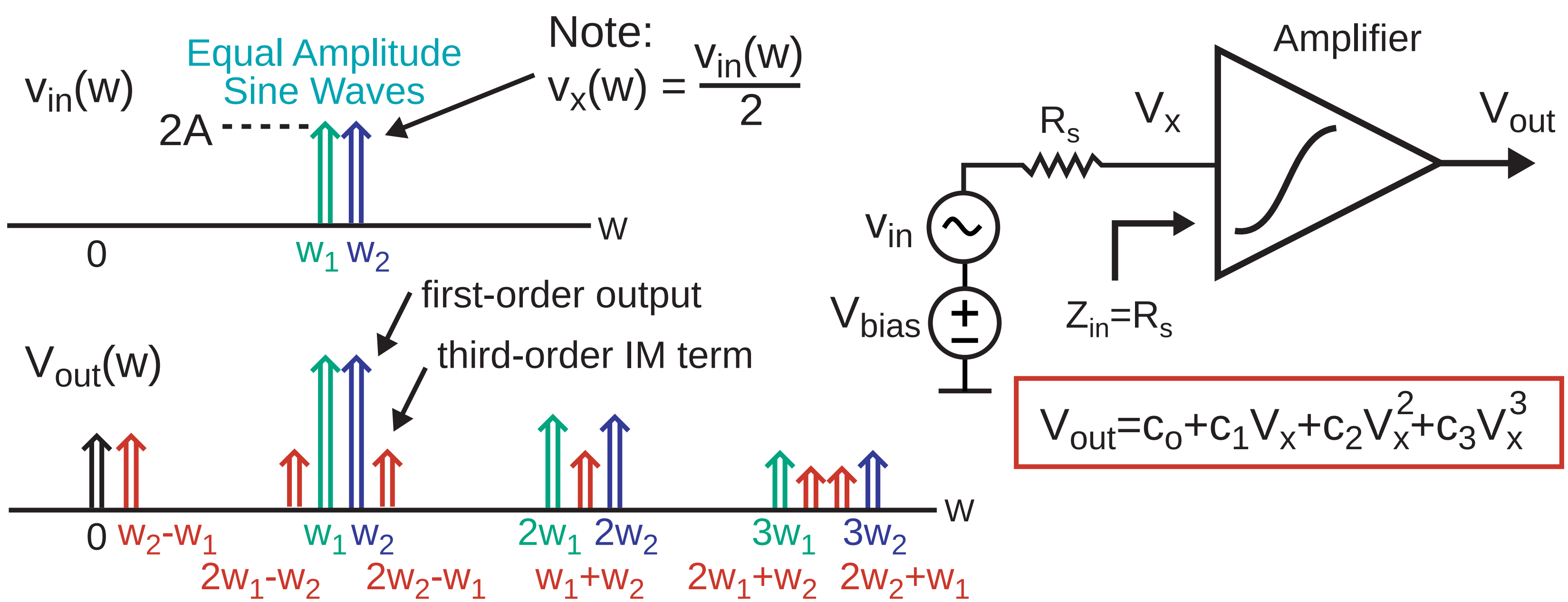
- **Magnitude of first order term is set by c_1 and A (for small A)**

$$(c_1A+\frac{9}{4}c_3A^3)(\cos w_1t+\cos w_2t) \approx c_1A(\cos w_1t+\cos w_2t)$$

- **Relative impact of intermodulation products can be calculated once we know A and the ratio of c_3 to c_1**
  - **Problem: it's often hard to extract the polynomial coefficients through direct DC measurements**
    - Need an indirect way to measure the ratio of c_3 to c_1

Two Tone Test

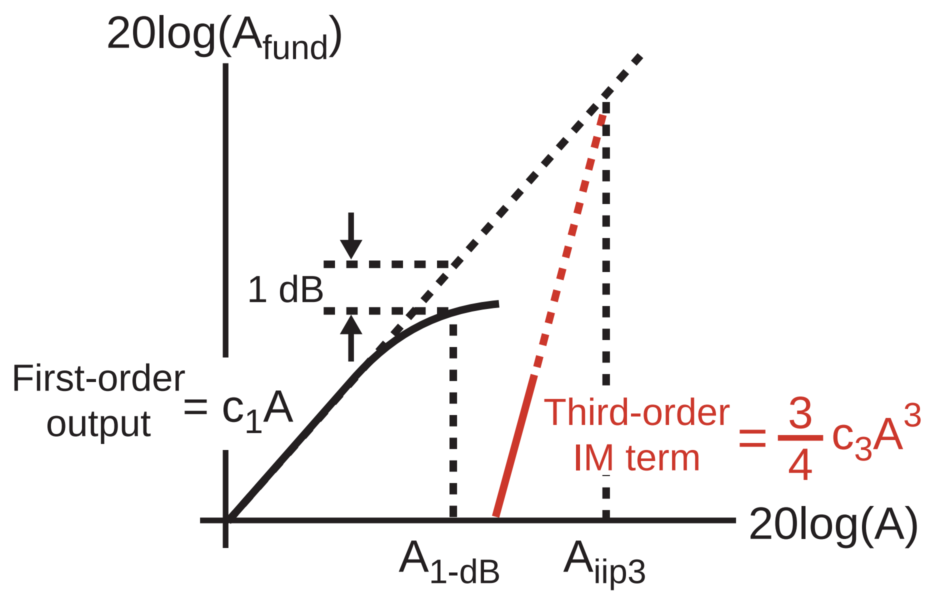
- **Input the sum of two equal amplitude sine waves into the amplifier (assume Z_{in} of amplifier = R_s of source)**



- **On a spectrum analyzer, measure first order and third order terms as *A* is varied (*A* must remain small)**
  - **First order term will increase linearly**
  - **Third order IM term will increase as the cube of *A***

Input-Referred Third Order Intercept Point (IIP3)

- **Plot the results of the two-tone test over a range of *A* (where *A* remains small) on a log scale (i.e., dB)**
  - **Extrapolate the results to find the intersection of the first and third order terms**



  - **IIP3 defined as the input power at which the extrapolated lines intersect (higher value is better)**
    - Note that IIP3 is a small signal parameter based on extrapolation, in contrast to the 1-dB compression point

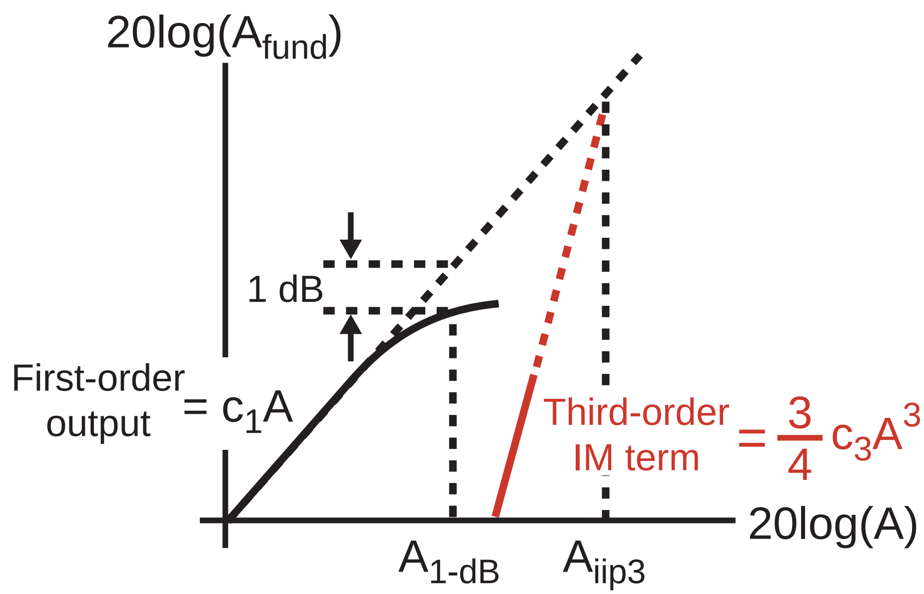
Relationship between IIP3, c_1 and c_3

- **Intersection point**

$$|c_1 A| = \left|\frac{3}{4} c_3 A^3\right|$$

- **Solve for A (gives A_{iip3})**

$$\Rightarrow A^2 = \frac{4}{3}\left|\frac{c_1}{c_3}\right| \quad (V_p^2)$$



- **Note that A corresponds to the peak value of the two cosine waves coming into the amplifier input node (V_x)**
  - **Would like to instead like to express IIP3 in terms of power**

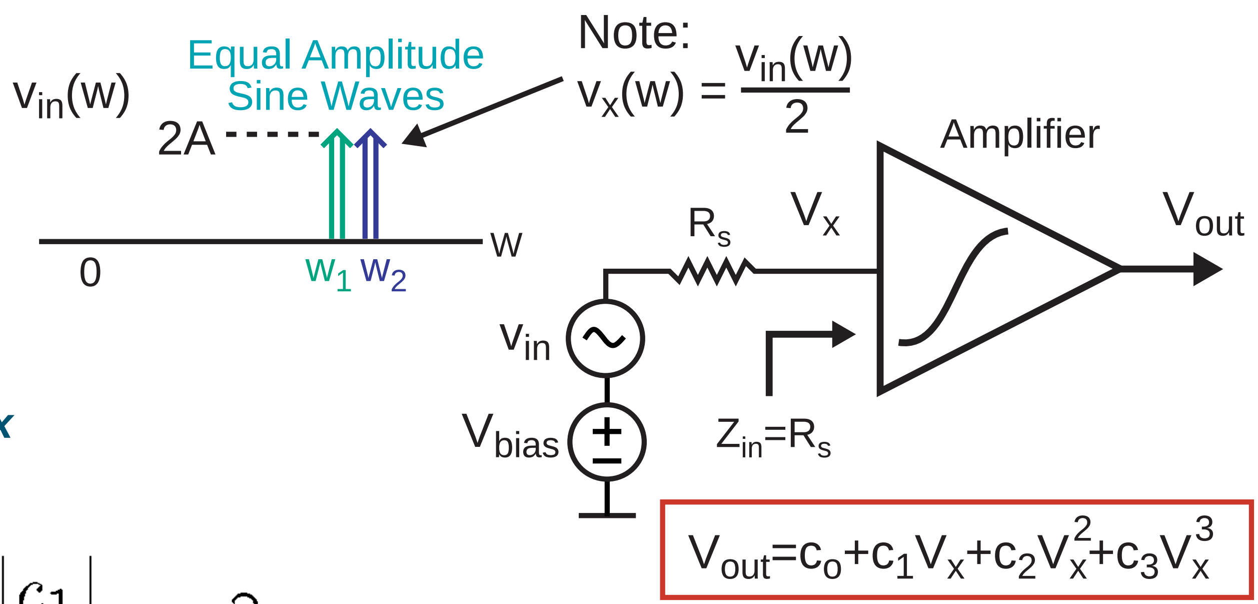
IIP3 Expressed in Terms of Power at Source

- **IIP3 referenced to V_x (peak voltage)**

$$A^2 = \frac{4}{3}\left|\frac{c_1}{c_3}\right| \quad (V_p^2)$$

- **IIP3 referenced to V_x (rms voltage)**

$$A_{rms}^2 = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{2}{3}\left|\frac{c_1}{c_3}\right| \quad (V_{rms}^2)$$



- **Power across $Z_{in} = R_s$**

$$\frac{A_{rms}^2}{R_s} = \frac{2}{3}\left|\frac{c_1}{c_3}\right|\frac{1}{R_s} \quad (Watts)$$

- **Note: Power from v_{in}**

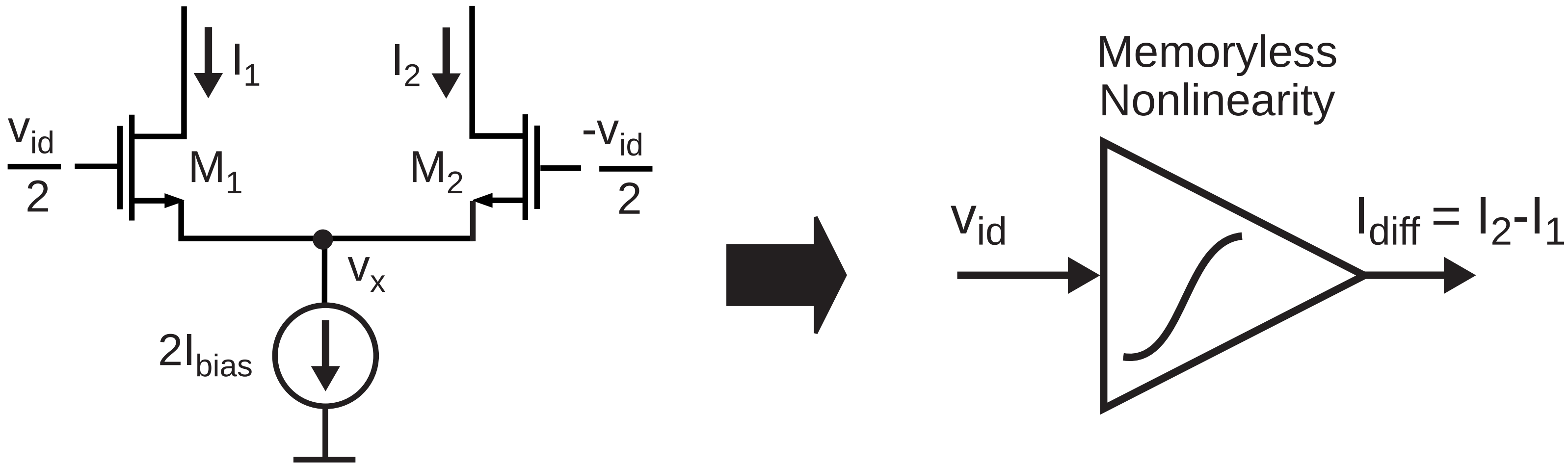
$$2\frac{A_{rms}^2}{R_s} = \frac{4}{3}\left|\frac{c_1}{c_3}\right|\frac{1}{R_s} \quad (Watts)$$

$$\Rightarrow IIP3(dBm) = 10\log\left((10^3)\frac{2}{3}\left|\frac{c_1}{c_3}\right|\frac{1}{R_s}\right) \quad (\text{dBm})$$

IIP3 as a Benchmark Specification

- **Since IIP3 is a convenient parameter to describe the level of third order nonlinearity in an amplifier, it is often quoted as a benchmark spec**
- **Measurement of IIP3 on a discrete amplifier would be done using the two-tone method described earlier**
  - **This is rarely done on integrated amplifiers due to poor access to the key nodes**
  - **Instead, for a radio receiver for instance, one would simply put in interferers and see how the receiver does**
    - Note: performance in the presence of interferers is not just a function of the amplifier nonlinearity
- **Calculation of IIP3 is most easily done using a Spice simulator**
  - **Two-tone method is not necessary – simply curve fit to a third order polynomial**

Impact of Differential Amplifiers on Nonlinearity



- **Assume v_x is approximately incremental ground**

$$I_{diff} = c_o + c_1\frac{v_{id}}{2} + c_2\left(\frac{v_{id}}{2}\right)^2 + c_3\left(\frac{v_{id}}{2}\right)^3$$
$$- \left(c_o + c_1\frac{-v_{id}}{2} + c_2\left(\frac{-v_{id}}{2}\right)^2 + c_3\left(\frac{-v_{id}}{2}\right)^3\right)$$

$$\Rightarrow \quad I_{diff} = c_1 v_{id} + \frac{c_3}{4}v_{id}^3$$

- **Second order term removed and IIP3 improved!**

Summary

- **Mismatch between devices in differential pair circuits induces an effective offset voltage**
  - **The value of the offset voltage is reduced by having large device dimensions**
  - **Fabrication reports or “Monte-Carlo” models provide the best approach to assessing the impact of mismatch**
    - May not be available, which leads to guessing the impact
- **Nonlinearity is typically modeled as a third order polynomial**
  - **Results in harmonic distortion and intermodulation**
  - **Third order component is often focused on in classical communication systems**
  - **Second order component is important for modern communication systems based on “direct conversion”**
  - **Differential pair offers some linearity advantages over single ended amplifiers**