Analysis and Design of Analog Integrated Circuits
Lecture 10

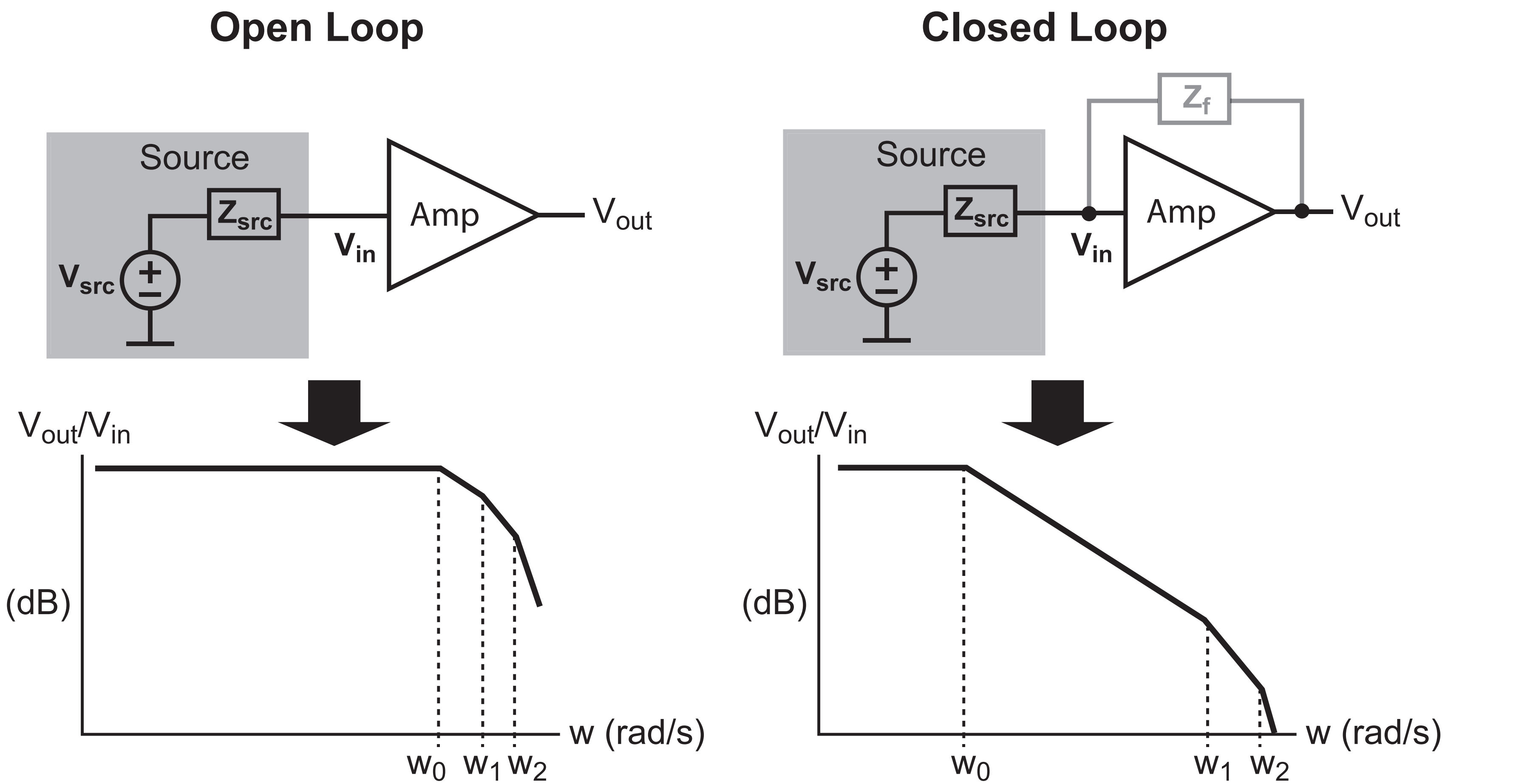
Frequency Response of Amplifiers

Michael H. Perrott

February 29, 2012

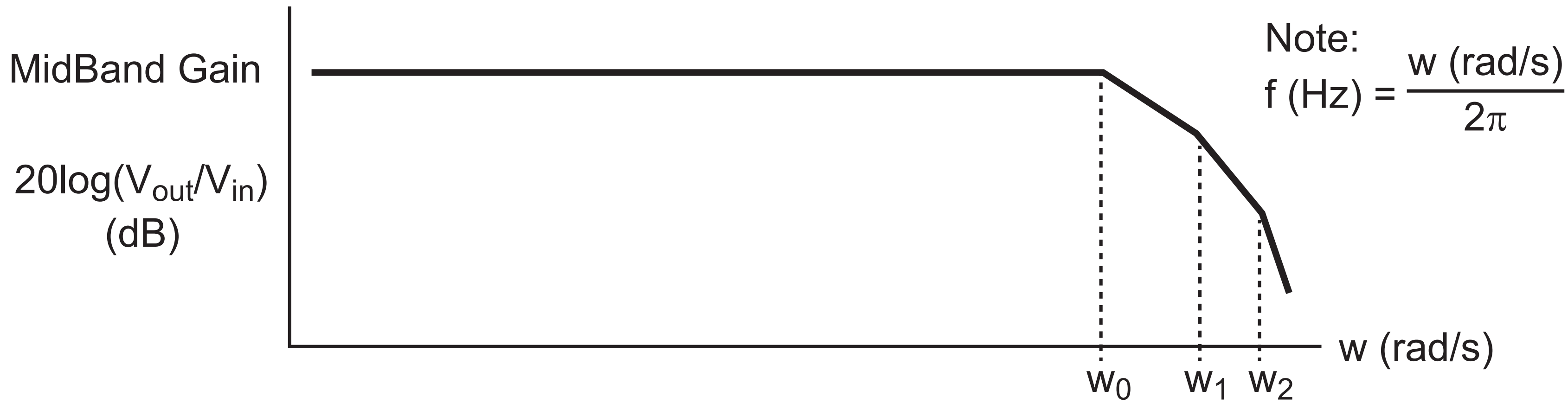
Copyright © 2012 by Michael H. Perrott
All rights reserved.

Open Loop Versus Closed Loop Amplifier Topologies



- **Open loop – want all bandwidth limiting poles to be as high in frequency as possible**
- **Closed loop – want one pole to be dominant and all other parasitic poles to be as high in frequency as possible**

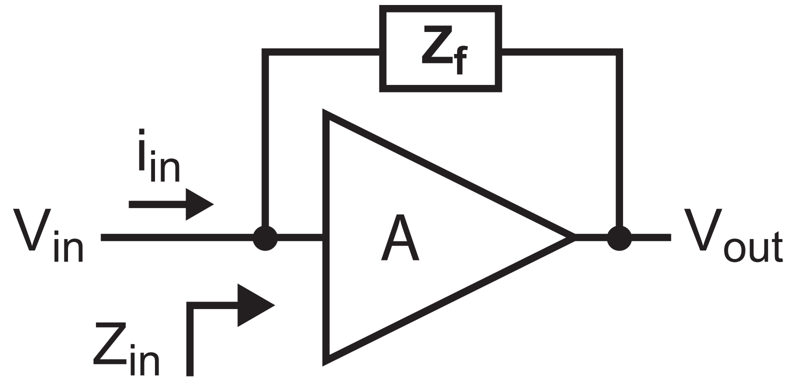
OCT Method of Estimating Amplifier Bandwidth



- **OCT method calculates $\sum_{i=0}^{n-1} \tau_i$ by the following steps:**
  - **Compute the effective resistance R_{thj} seen by each capacitor, C_j, with other caps as open circuits**
    - AC coupling caps are not included – considered as shorts
  - **Form the "open circuit" time constant $T_j = R_{thj}C_j$ for each capacitor C_j**
  - **Sum all of the "open circuit" time constants**

$$\Rightarrow \; BW \approx \frac{1}{\sum_{j=1}^{m} R_{thj} C_j} \; rad/s$$

Another Useful Analysis Tool: Miller Effect



- **Derive input impedance (assume gain of amplifier = *A*):**

$$Z_{in} = \frac{V_{in}}{i_{in}} = \frac{V_{in}}{(V_{in} - V_{out})/Z_f} = \frac{V_{in}Z_f}{V_{in} - AV_{in}} = \frac{Z_f}{1 - A}$$

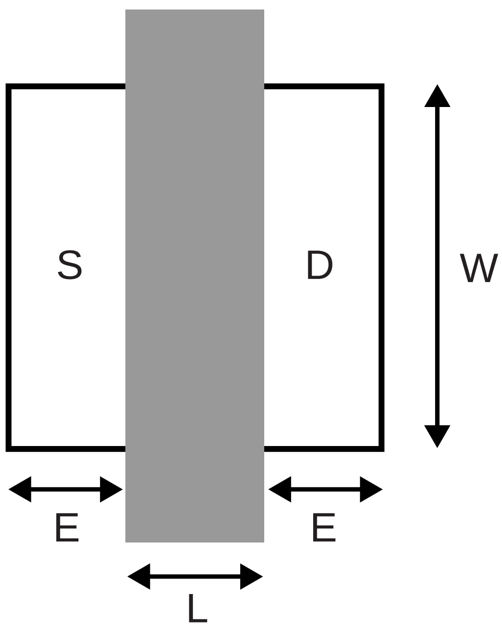
- **Consider the case where Z_f is a capacitor**

$$Z_f = \frac{1}{sC} \Rightarrow Z_{in} = \frac{1}{s(1-A)C}$$

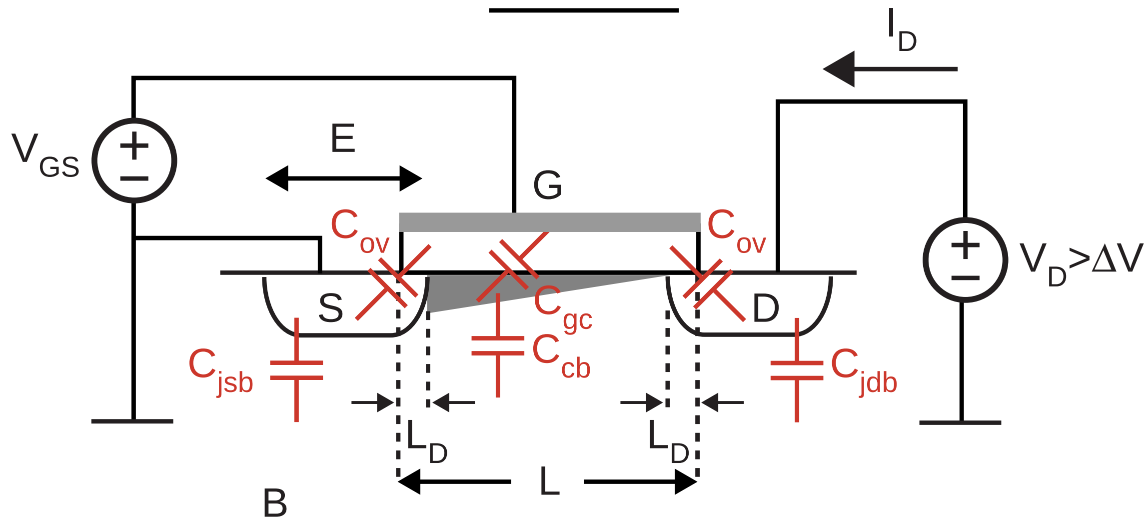
  - **For negative *A*, input impedance sees increased cap value**
  - **For *A* = 1, input impedance sees no influence from cap**
  - **For *A* > 1, input impedance sees negative capacitance!**
    - Can be used to create active inductor for a specific frequency

Key Capacitances for CMOS Devices

Top View



Side View



junction bottom wall cap (per area)

junction sidewall cap (per length)

source to bulk cap: $C_{jsb} = \dfrac{C_j(0)}{\sqrt{1 + V_{SB}/|\Phi_B|}} WE + \dfrac{C_{jsw}(0)}{\sqrt{1 + V_{SB}/|\Phi_B|}} (W + 2E)$

(make 2W for "4 sided" perimeter in some cases)

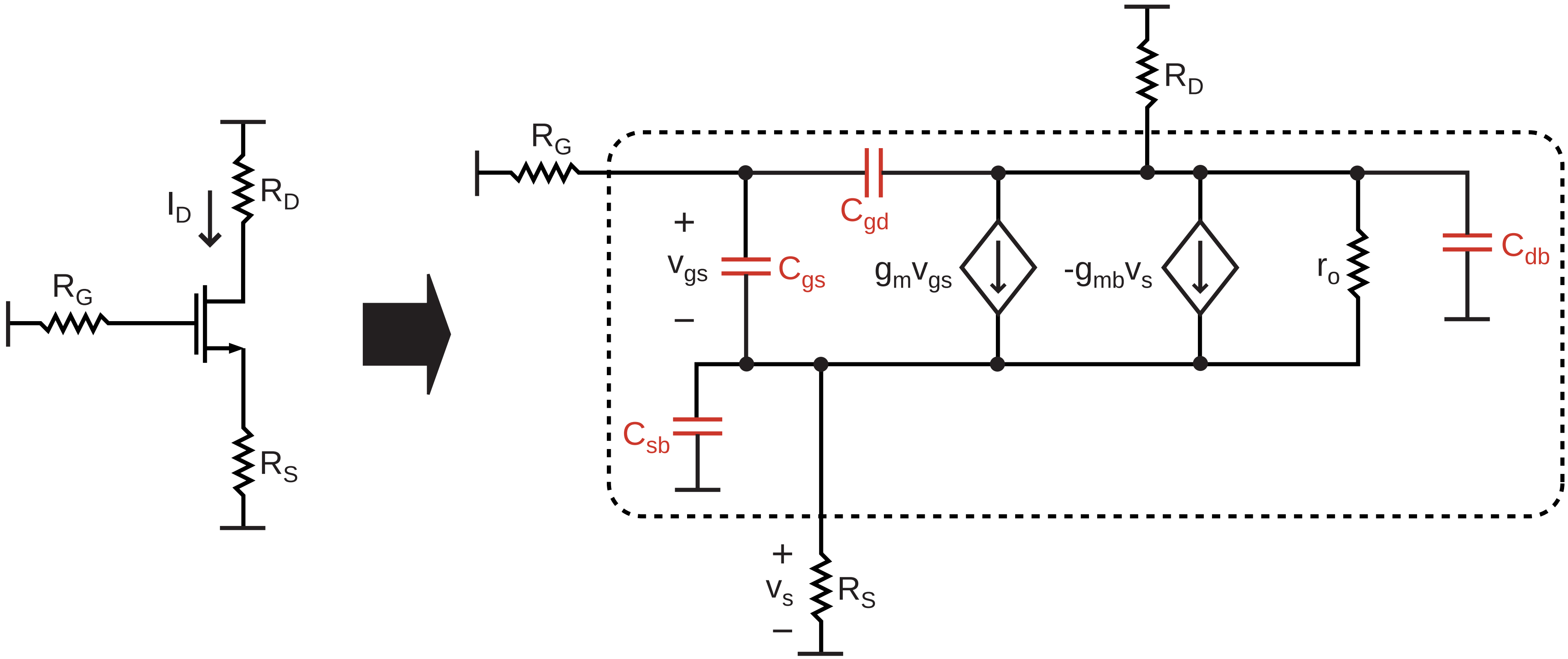
drain to bulk cap: $C_{jsd} = \dfrac{C_j(0)}{\sqrt{1 + V_{DB}/|\Phi_B|}} WE + \dfrac{C_{jsw}(0)}{\sqrt{1 + V_{DB}/|\Phi_B|}} (W + 2E)$

overlap cap: $C_{ov} = WL_D C_{ox} + WC_{fringe}$

gate to channel cap: $C_{gc} = \dfrac{2}{3} C_{ox} W(L - 2L_D)$

channel to bulk cap: C_{cb} - ignore in this class

CMOS Hybrid-π Model with Caps (Device in Saturation)



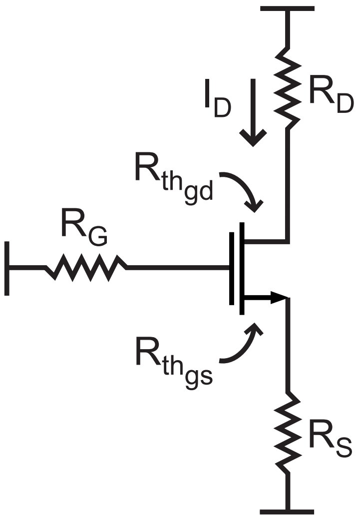
$$C_{gs} = C_{gc} + C_{ov} = \frac{2}{3} C_{ox} W(L\text{-}2L_D) + C_{ov}$$

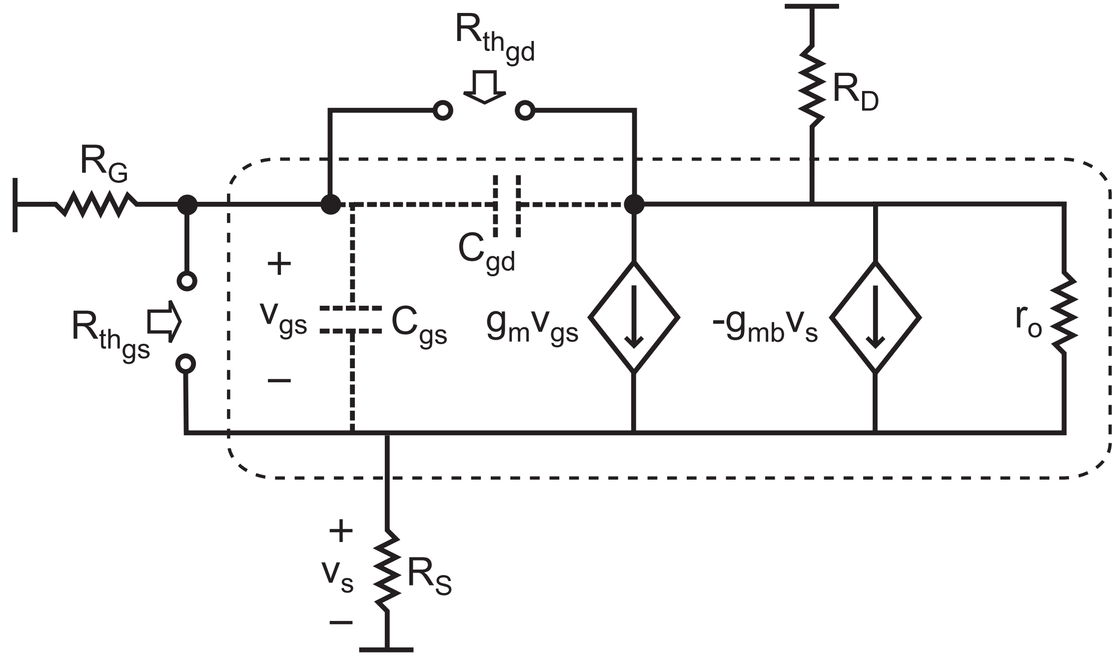
$$C_{gd} = C_{ov}$$

$C_{sb} = C_{jsb}$ (area + perimeter junction capacitance)

$C_{db} = C_{jdb}$ (area + perimeter junction capacitance)

OCT Thevenin Resistance Calculations





- **C_{gs}: Thevenin resistance between gate and source**

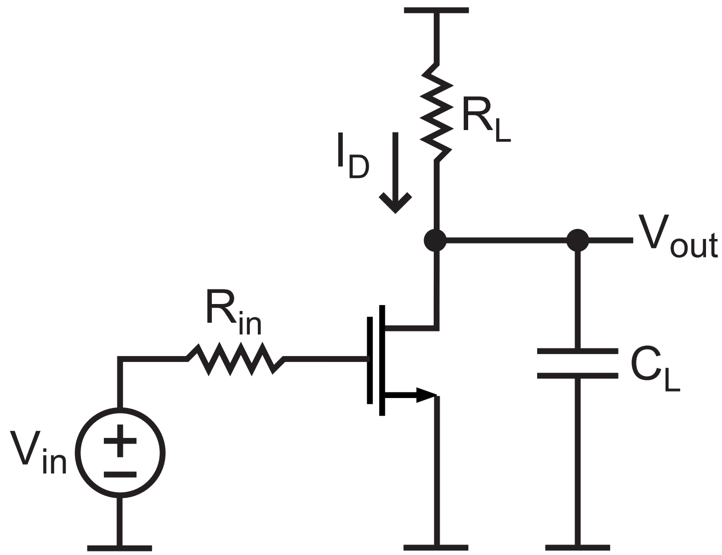
$$R_{th_{gs}} = \frac{R_S(1 + R_D/r_o) + R_G(1 + (g_{mb} + 1/r_o)R_S + R_D/r_o)}{1 + (g_m + g_{mb})R_S + (R_S + R_D)/r_o}$$

- **C_{gd}: Thevenin resistance between gate and drain**

$$R_{th_{gd}} = (R_D + R_G)(1 - r_{ods}/r_o) + r_{ods} g_m R_G$$

$$\text{where } r_{ods} = r_o || \frac{R_D}{1 + (g_m + g_{mb})R_S}$$

OCT Example: Design Wide Bandwidth Amplifier

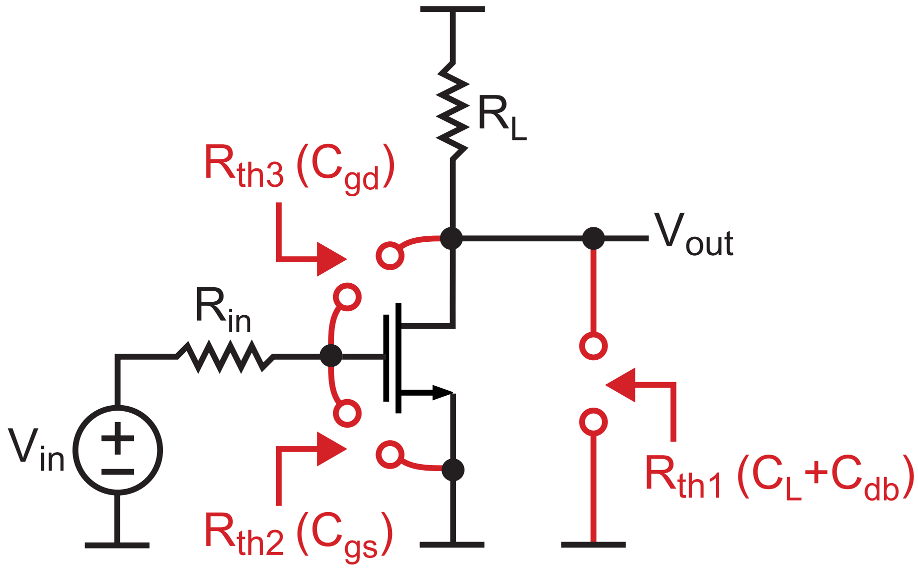


Assumptions:

g_m = 1/(100Ω), γ = 0, λ = 0
C_{gs} = 10fF, C_{gd} = 3fF
C_{sb} = 5fF, C_{db} = 4fF
R_{in} = 4kΩ
R_L = 1kΩ
C_L = 100fF

- **Step 1: identify AC coupling versus OCT capacitors**
  - **AC coupling caps will be regarded as shorts**
- **Step 2: calculate individual OCT time constants**
- **Step 3: identify long OCT time constants and modify circuit to improve its bandwidth**

Step 1: Identify OCT Capacitors



<u>Assumptions:</u>

g_m = 1/(100Ω), γ = 0, λ = 0

C_{gs} = 10fF, C_{gd} = 3fF

C_{sb} = 5fF, C_{db} = 4fF

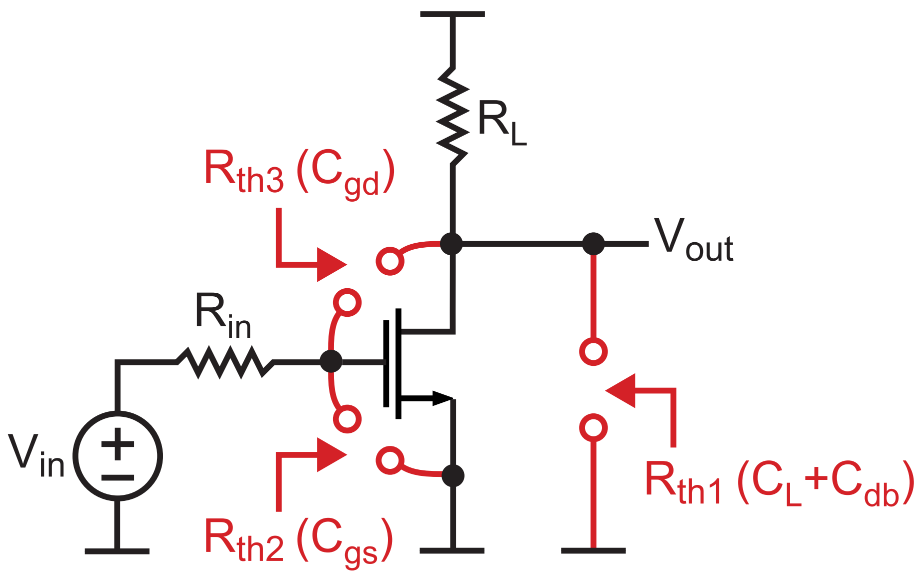
R_{in} = 4kΩ

R_L = 1kΩ

C_L = 100fF

- **Which time constants are easy to calculate?**
- **How do we efficiently calculate the more difficult cases?**

Step 2: OCT Time Constant Calculations



Assumptions:

g_m = 1/(100Ω), γ = 0, λ = 0
C_{gs} = 10fF, C_{gd} = 3fF
C_{sb} = 5fF, C_{db} = 4fF
R_{in} = 4kΩ
R_L = 1kΩ
C_L = 100fF

- **Easy ones:**

$$R_{th1} = R_L || R_{th_d} = R_L || \infty = R_L = 1k\Omega \;\Rightarrow\; \tau_1 = 1k\Omega \cdot 104fF = \boxed{104ps}$$

$$R_{th2} = R_{in} || R_{th_g} = R_{in} || \infty = R_{in} = 4k\Omega \;\Rightarrow\; \tau_2 = 4k\Omega \cdot 10fF = \boxed{40ps}$$

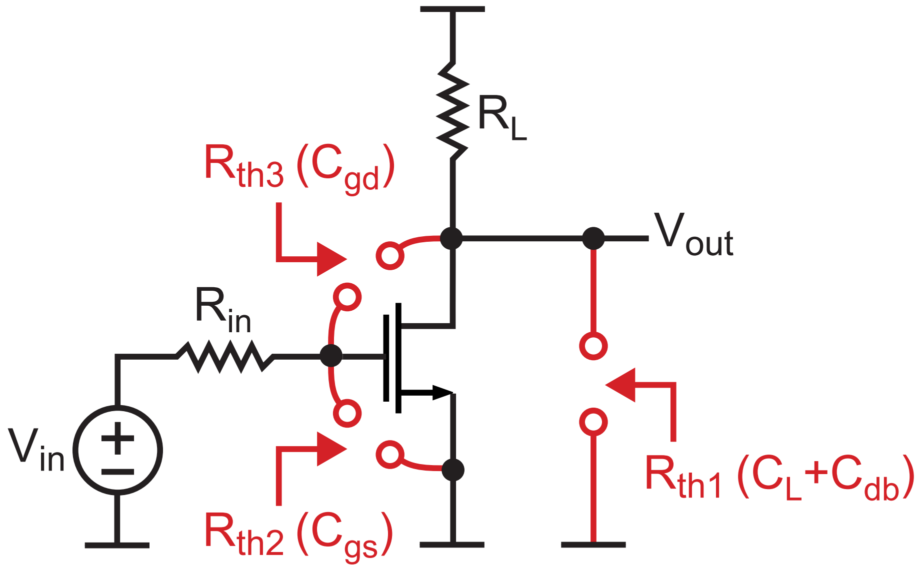
- **Use formula for τ_3:** $R_{th_{gd}} = (R_D + R_G)(1 - r_{ods}/r_o) + r_{ods} g_m R_G$

$$\text{where } r_{ods} = r_o || \frac{R_D}{1 + (g_m + g_{mb})R_S} = R_D = R_L$$

$$\Rightarrow \; R_{th3} = (R_L + R_{in})(1 - 0) + R_L g_m R_{in} = 5.5k\Omega + 40k\Omega = 45.5k\Omega$$

$$\Rightarrow \; \tau_3 = 45.5k\Omega \cdot 3fF = \boxed{136.5ps}$$

Step 3: Identify Largest OCT Time Constant



Assumptions:

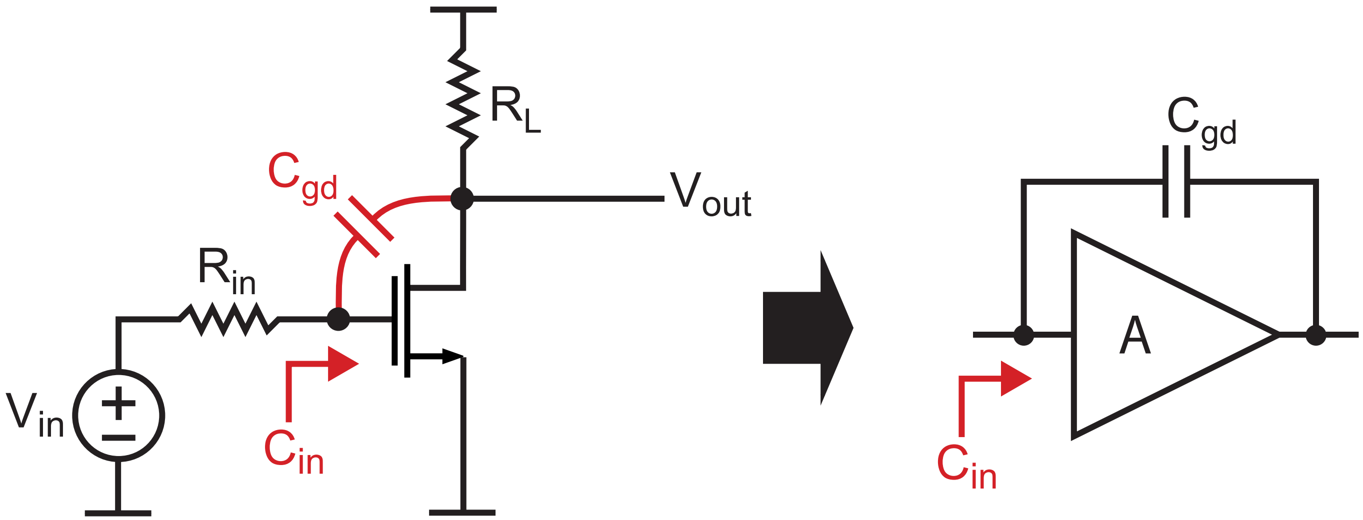
$g_m = 1/(100\Omega)$, $\gamma = 0$, $\lambda = 0$
$C_{gs} = 10fF$, $C_{gd} = 3fF$
$C_{sb} = 5fF$, $C_{db} = 4fF$
$R_{in} = 4k\Omega$
$R_L = 1k\Omega$
$C_L = 100fF$

- **Time constant associated with C_{gd} is the longest:**

$$\tau_3 = 45.5k\Omega \cdot 3fF = \boxed{136.5ps}$$

- **Why is this time constant so large given that it is associated with the lowest value capacitor?**
- **How do we change the amplifier topology to reduce this time constant value?**

The Miller Effect Analysis Provides Helpful Intuition



- **Notice that C_{gd} is in the feedback path of the common source amplifier**
  - **Recall Miller effect calculation:** $C_{in} = (1 - A)C_{gd}$
  - **For this amplifier:**

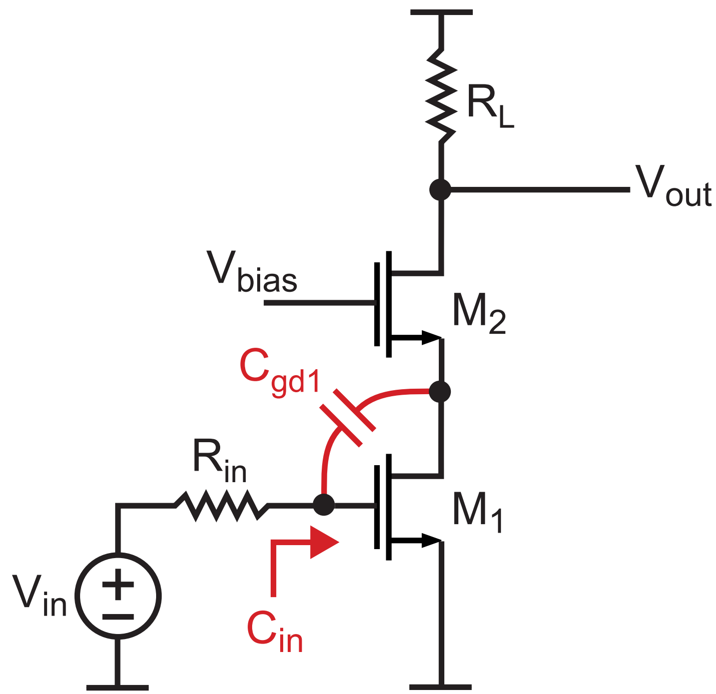
$$A = -g_m R_L \;\; \Rightarrow \;\; C_{in} = (1 + g_m R_L)C_{gd} = 11 \cdot C_{gd} = 33fF$$

$$\Rightarrow \;\; \tau_3 = R_{in}C_{in} = 4k\Omega \cdot 33fF \; = \; 132ps$$

- This analysis agrees well with OCT calculation of 136.5ps

Can we change the amplifier topology to lower this time constant?

Consider Adding a Cascode Device



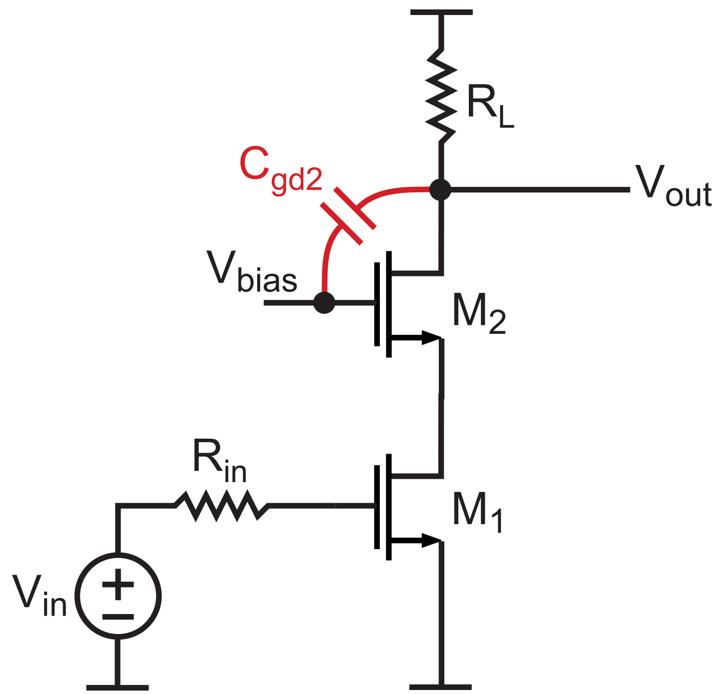
- **Examine the impact of this topological change using the Miller Effect analysis**

$$A = -g_{m1}\frac{1}{g_{m2}} \approx -1 \quad \Rightarrow \quad C_{in} = (1+1)C_{gd1} = 2 \cdot C_{gd1} = 6fF$$

$$\Rightarrow \quad \tau_3 = R_{in}C_{in} = 4k\Omega \cdot 6fF \; = \; 24ps$$

Cascode device dramatically reduces the C_{gd1} time constant!

Does the Miller Effect Impact the Cascode Device?

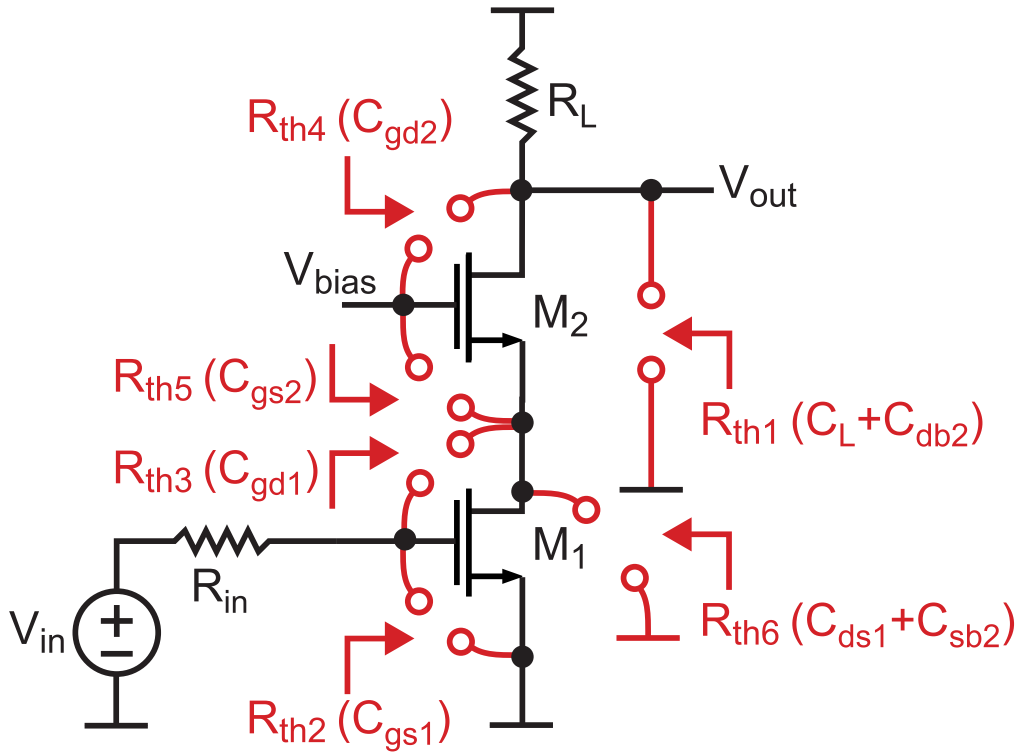


- **Observe that the capacitance seen by V_{bias} is not of concern since this voltage is not part of the signal path**
- **The signal path sees the time constant:**

$$\tau_4 = R_L || R_{th_{d2}} \cdot C_{gd2} \approx R_L \cdot C_{gd2} = 1k\Omega \cdot 3fF = 3ps$$

  - **This time constant is much smaller than the other time constants of the amplifier**

Perform OCT Calculations for Updated Amplifier



Assumptions for all devices:

$g_m = 1/(100\Omega)$, $\gamma = 0$, $\lambda = 0$
$C_{gs} = 10fF$, $C_{gd} = 3fF$
$C_{sb} = 5fF$, $C_{db} = 4fF$
$R_{in} = 4k\Omega$
$R_L = 1k\Omega$
$C_L = 100fF$

$$R_{th1} = R_L || R_{th_{d2}} = R_L = 1k\Omega \Rightarrow \tau_1 = 1k\Omega \cdot 104fF = \boxed{104ps}$$

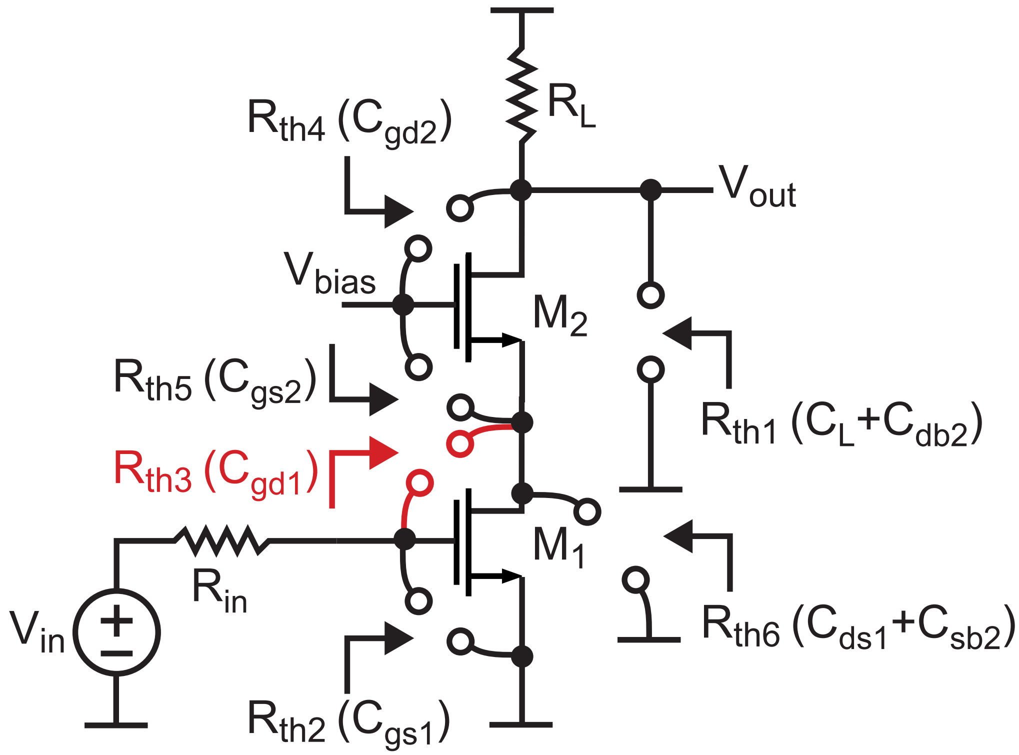
$$R_{th2} = R_{in} || R_{th_{g1}} = R_{in} = 4k\Omega \Rightarrow \tau_2 = 4k\Omega \cdot 10fF = \boxed{40ps}$$

$$R_{th4} = R_L || R_{th_{d2}} \approx R_L = 1k\Omega \Rightarrow \tau_3 = 1k\Omega \cdot 3fF = \boxed{3ps}$$

$$R_{th5} = R_{th_{s2}} || R_{th_{d1}} \approx 1/g_{m2} || \infty = 100\Omega \Rightarrow \tau_5 = 100\Omega \cdot 10fF = \boxed{1ps}$$

$$R_{th6} = R_{th_{d1}} || R_{th_{s2}} = \infty || 1/g_{m2} = 100\Omega \Rightarrow \tau_6 = 100\Omega \cdot 9fF = \boxed{0.9ps}$$

Perform OCT Calculations for Updated Amplifier



Assumptions for all devices:

$g_m = 1/(100\Omega)$, $\gamma = 0$, $\lambda = 0$
$C_{gs} = 10fF$, $C_{gd} = 3fF$
$C_{sb} = 5fF$, $C_{db} = 4fF$
$R_{in} = 4k\Omega$
$R_L = 1k\Omega$
$C_L = 100fF$

- **Use Thevenin formula for C_{gd} calculation:**

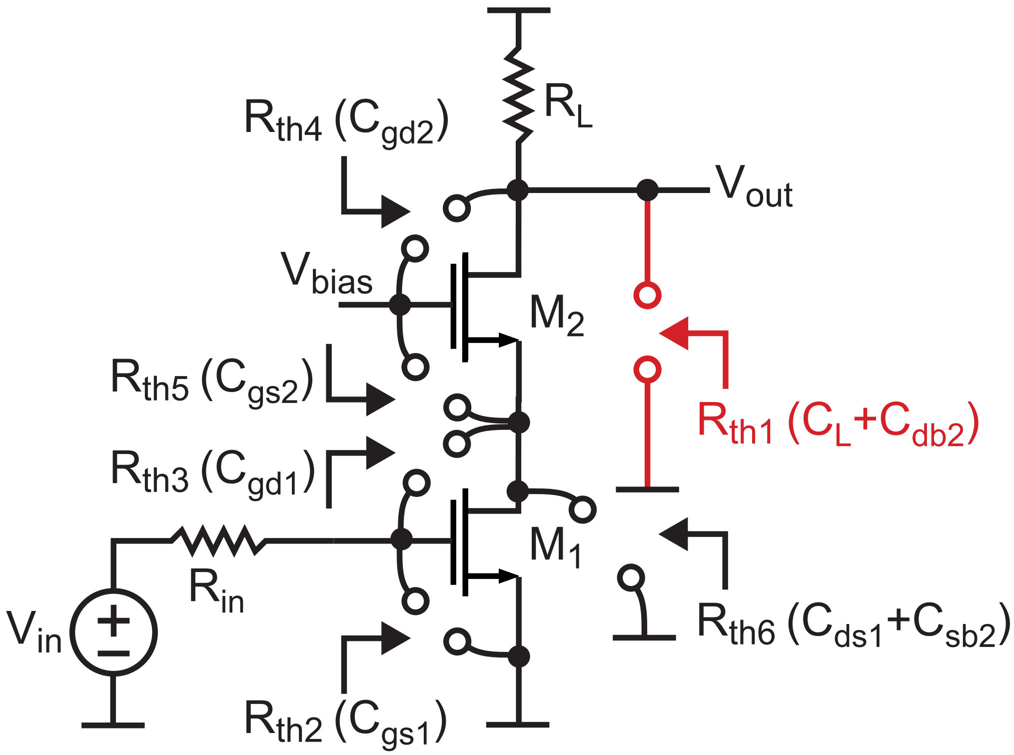
$$R_{th3} = (R_{D1} + R_{G1})(1 - r_{ods}/r_{o1}) + r_{ods} g_{m1} R_{G1}$$

$$\text{where } r_{ods} = r_{o1} || \frac{R_{D1}}{1 + (g_{m1} + g_{mb1})R_{S1}}$$

$$\Rightarrow R_{th3} = (\frac{1}{g_{m2}} + R_{in})(1 - 0) + \frac{1}{g_{m2}} g_{m1} R_{in} = 4.1k\Omega + 4k\Omega = 8.1k\Omega$$

$$\Rightarrow \tau_3 = 8.1k\Omega \cdot 3fF = \boxed{24.3ps}$$

Identify Longest OCT Time Constant



Assumptions for all devices:

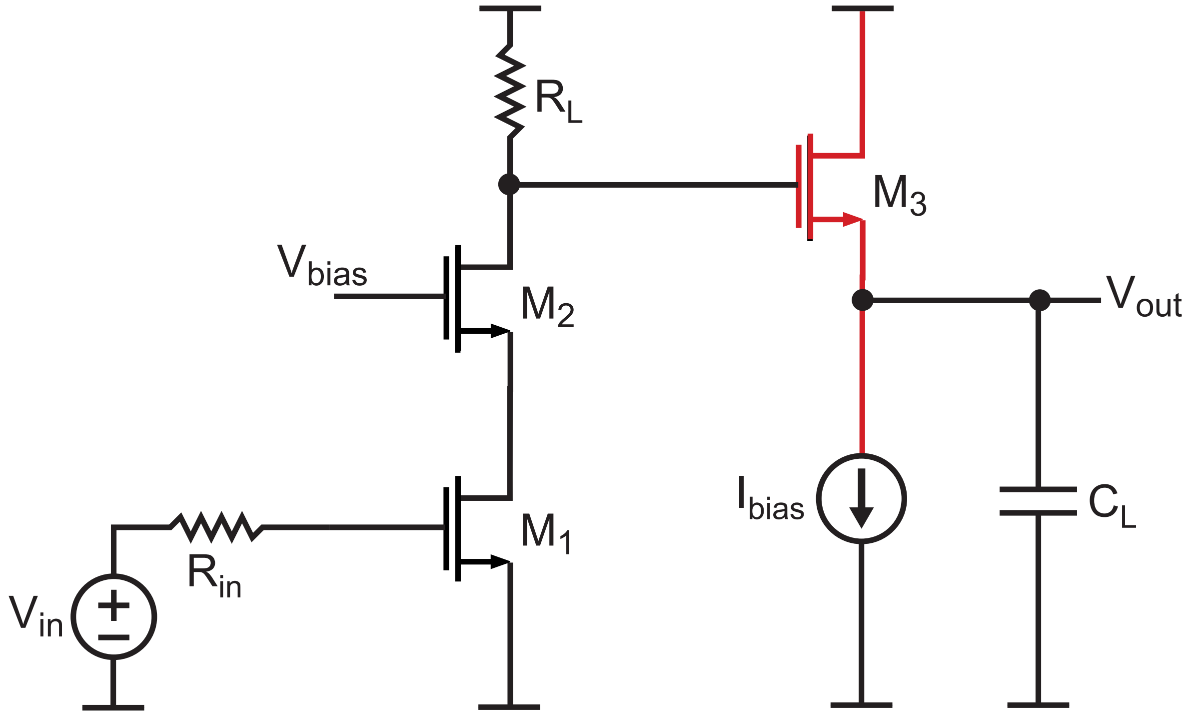
$g_m = 1/(100\Omega)$, $\gamma = 0$, $\lambda = 0$
$C_{gs} = 10fF$, $C_{gd} = 3fF$
$C_{sb} = 5fF$, $C_{db} = 4fF$
$R_{in} = 4k\Omega$
$R_L = 1k\Omega$
$C_L = 100fF$

- **The load capacitance now presents the largest time constant:**

$$R_{th1} = R_L || R_{th_{d2}} = R_L = 1k\Omega \;\Rightarrow\; \tau_1 = 1k\Omega \cdot 104fF = \boxed{104ps}$$

Can we change the amplifier topology to lower this time constant?

Add a Source Follower to the Output

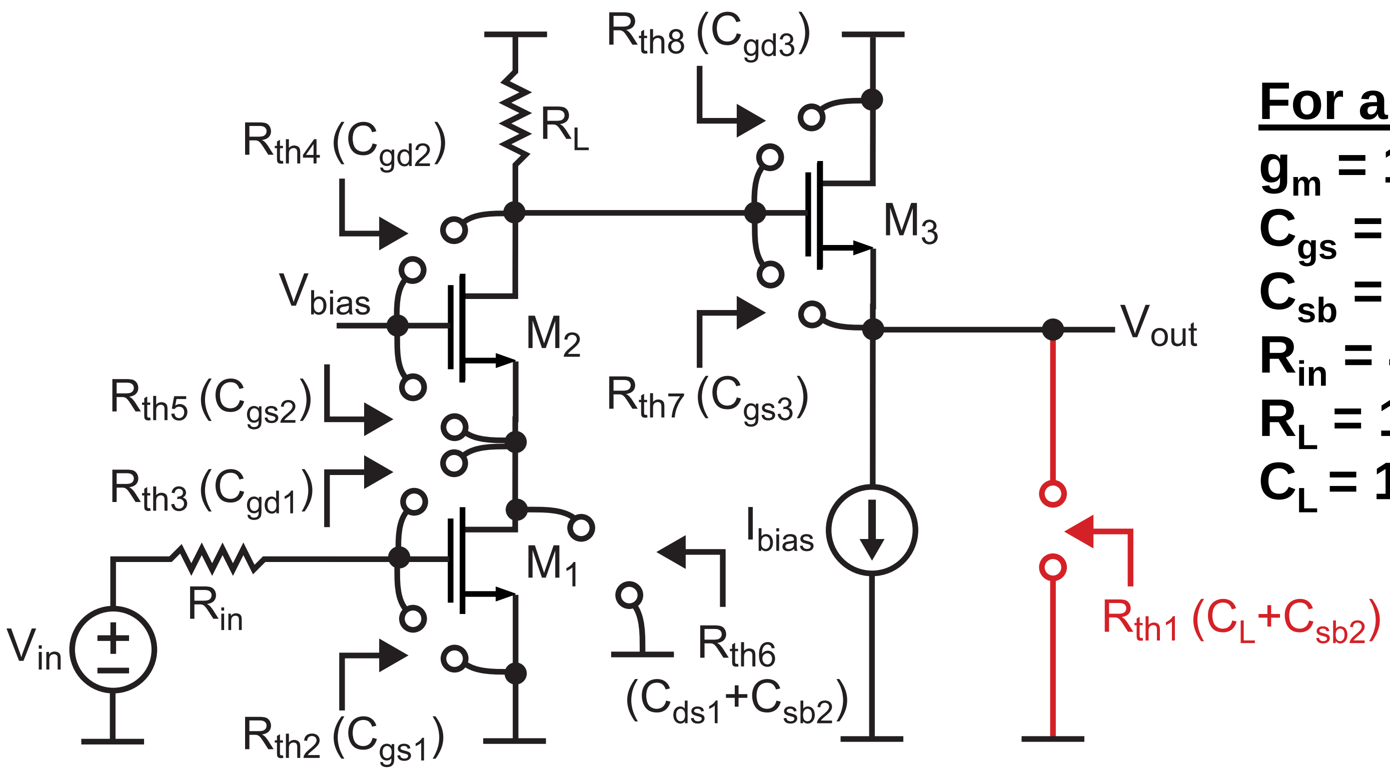


For all devices:
$g_m = 1/(100\Omega)$, $\gamma = 0$, $\lambda = 0$
$C_{gs} = 10fF$, $C_{gd} = 3fF$
$C_{sb} = 5fF$, $C_{db} = 4fF$
$R_{in} = 4k\Omega$
$R_L = 1k\Omega$
$C_L = 100fF$

- **Key idea: reduce the time constant associated with C_L by decreasing the Thevenin resistance that it sees**
  - **Previous design presented $R_L = 1K\Omega$ to C_L**
  - **Source follower presents $R_{ths3} = 1/g_{m3} = 100\Omega$ to C_L**

Source follower should reduce C_L time constant by a factor of ten!

Calculation of New C_L Time Constant



For all devices:

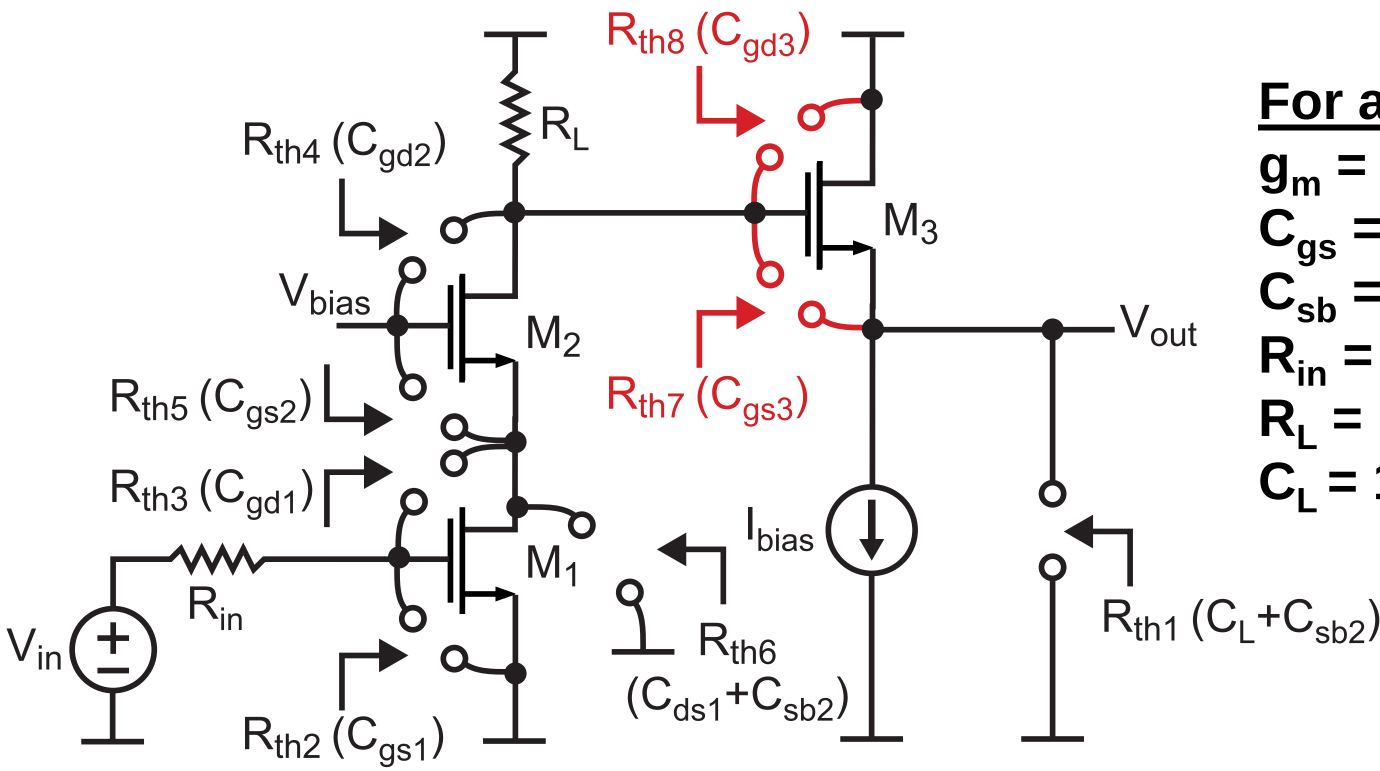
g_m = 1/(100Ω), γ = 0, λ = 0
C_{gs} = 10fF, C_{gd} = 3fF
C_{sb} = 5fF, C_{db} = 4fF
R_{in} = 4kΩ
R_L = 1kΩ
C_L = 100fF

- **Formal calculation:**

$$R_{th1} = R_{th_{s3}} = 1/g_{m3} = 100\Omega \;\Rightarrow\; \tau_1 = 100\Omega \cdot 104fF = \boxed{10.4ps}$$

How large are the additional time constants created by M_3?

Calculation of Additional Time Constants from M_3



For all devices:

$g_m = 1/(100\Omega)$, $\gamma = 0$, $\lambda = 0$

$C_{gs} = 10fF$, $C_{gd} = 3fF$

$C_{sb} = 5fF$, $C_{db} = 4fF$

$R_{in} = 4k\Omega$

$R_L = 1k\Omega$

$C_L = 100fF$

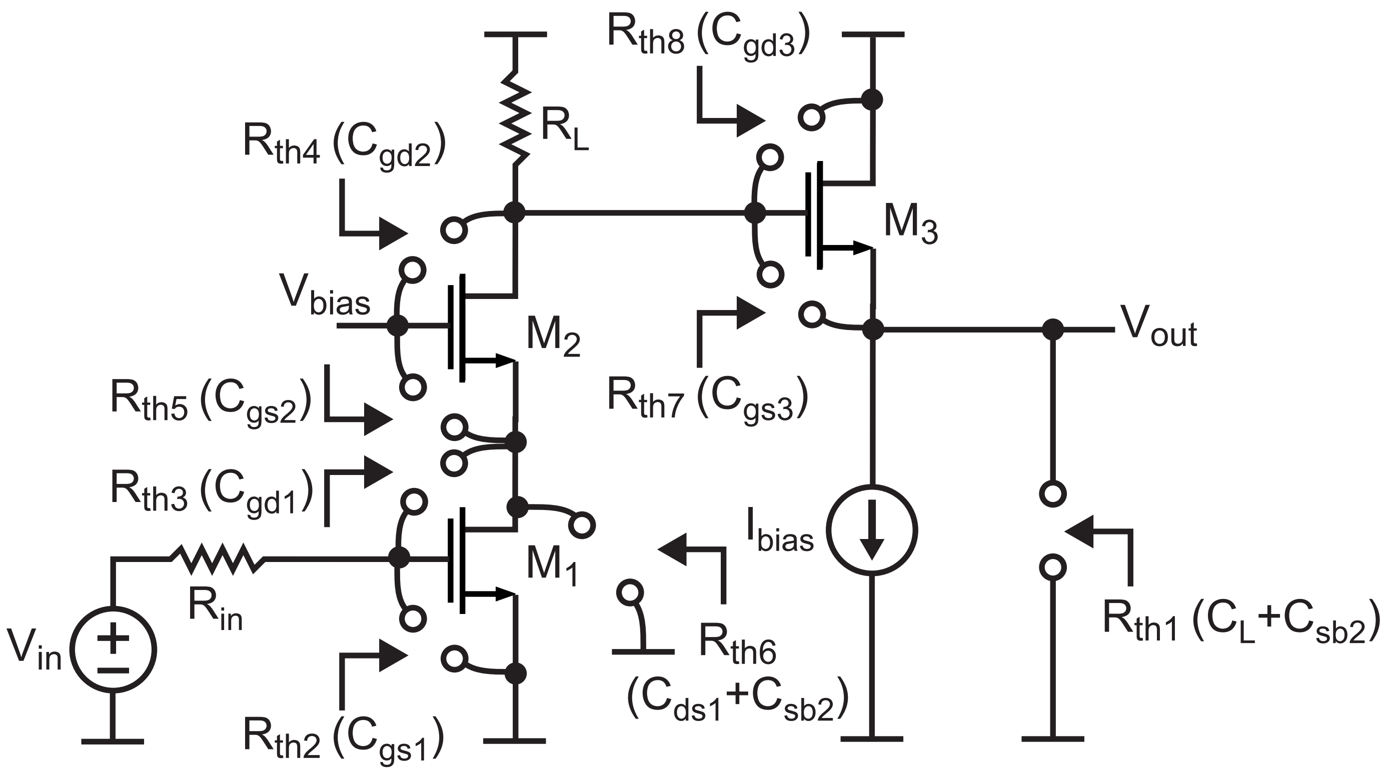
$$R_{th8} = R_L || R_{th_{d2}} \approx R_L = 1k\Omega \;\Rightarrow\; \tau_8 = 1k\Omega \cdot 3fF = \boxed{3ps}$$

$$R_{th7} = \frac{R_{S3}(1 + R_{D3}/r_{o3}) + R_{G3}(1 + (g_{mb3} + 1/r_{o3})R_{S3} + R_{D3}/r_{o3})}{1 + (g_{m3} + g_{mb3})R_{S3} + (R_{S3} + R_{D3})/r_{o3}}$$

$$\Rightarrow\; R_{th7} = \frac{1 + R_{D3}/r_{o3} + R_{G3}(g_{mb3} + 1/r_{o3})}{g_{m3} + g_{mb3} + 1/r_{o3}} = \frac{1 + 0 + 0}{g_{m3} + 0 + 0} = 100\Omega$$

$$\Rightarrow\; \tau_7 = 100\Omega \cdot 10fF = \boxed{1ps}$$

Estimate Bandwidth Based on OCT Calculations



$$\tau_1 = 10.4ps$$
$$\tau_2 = 40ps$$
$$\tau_3 = 24.3ps$$
$$\tau_4 = 3ps$$
$$\tau_5 = 1ps$$
$$\tau_6 = 0.9ps$$
$$\tau_7 = 1ps$$
$$\tau_8 = 3ps$$

$$BW \approx \frac{1}{\sum_{j=1}^{m} R_{thj}C_j} = \frac{1}{83.6ps} = 11.96\ Grad/s$$

$$\Rightarrow\ BW \approx \frac{11.96}{2\pi} = \boxed{1.9GHz}$$

Summary

- **Two techniques prove very useful when designing amplifiers for desired frequency response behavior**
  - **Open Circuit Time Constant method**
  - **Miller Effect analysis**
- **Thevenin resistance analysis in combination with the above offers tremendous insight for designing amplifier topologies**
  - **OCT method allows quick discovery of large time constants**
  - **Miller effect provides intuition of the impact of placing capacitors within feedback**
  - **Awareness of impedances presented by various amplifier stages allows intuitive approach to achieve reduction of large time constants**