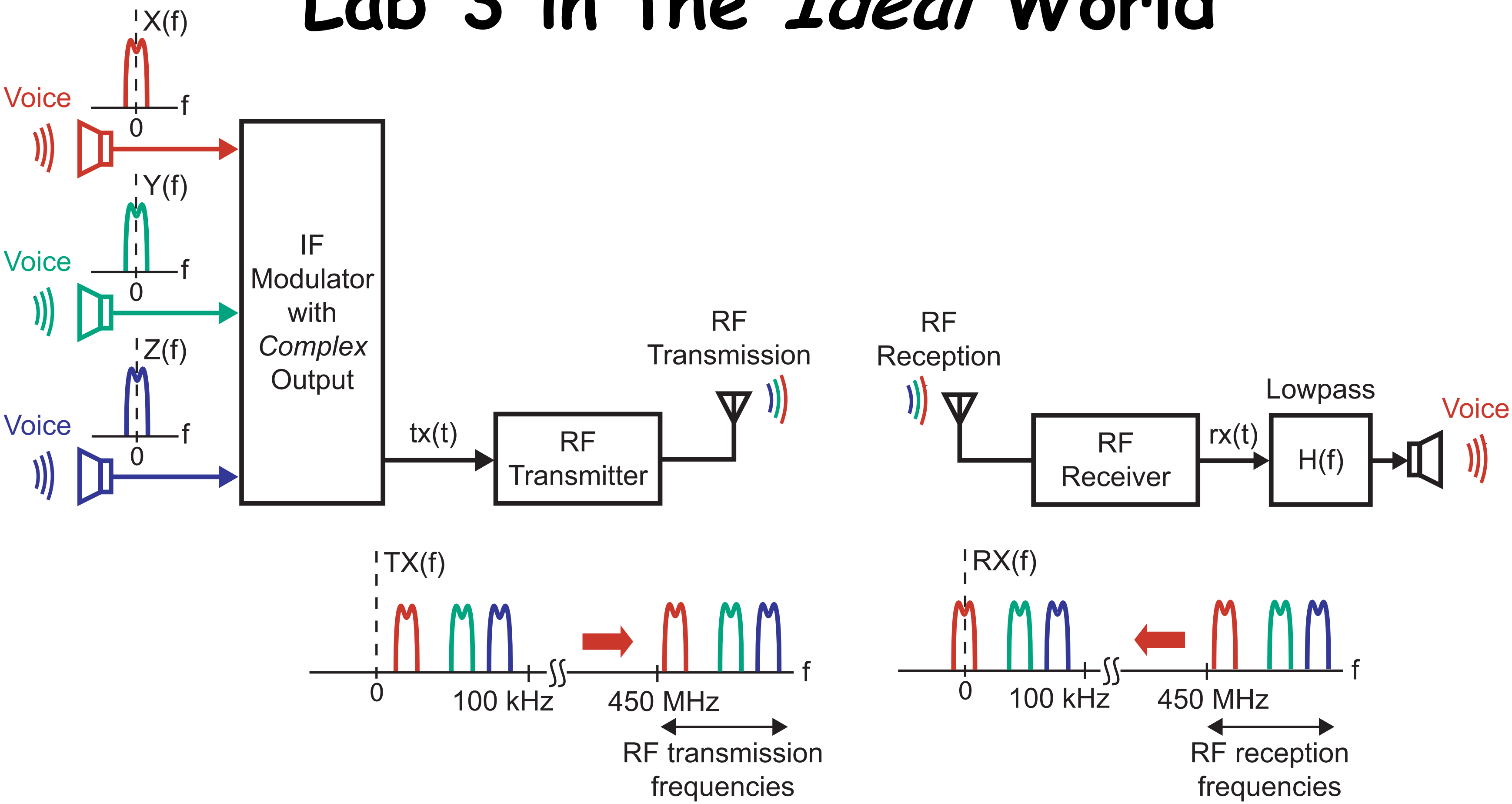
Energy and Noise

- Signal metrics: mean, power, energy
- Signal-to-Noise Ratio
- Random processes
- Probability Density Function, mean, variance

Lab 3 in the *Ideal* World

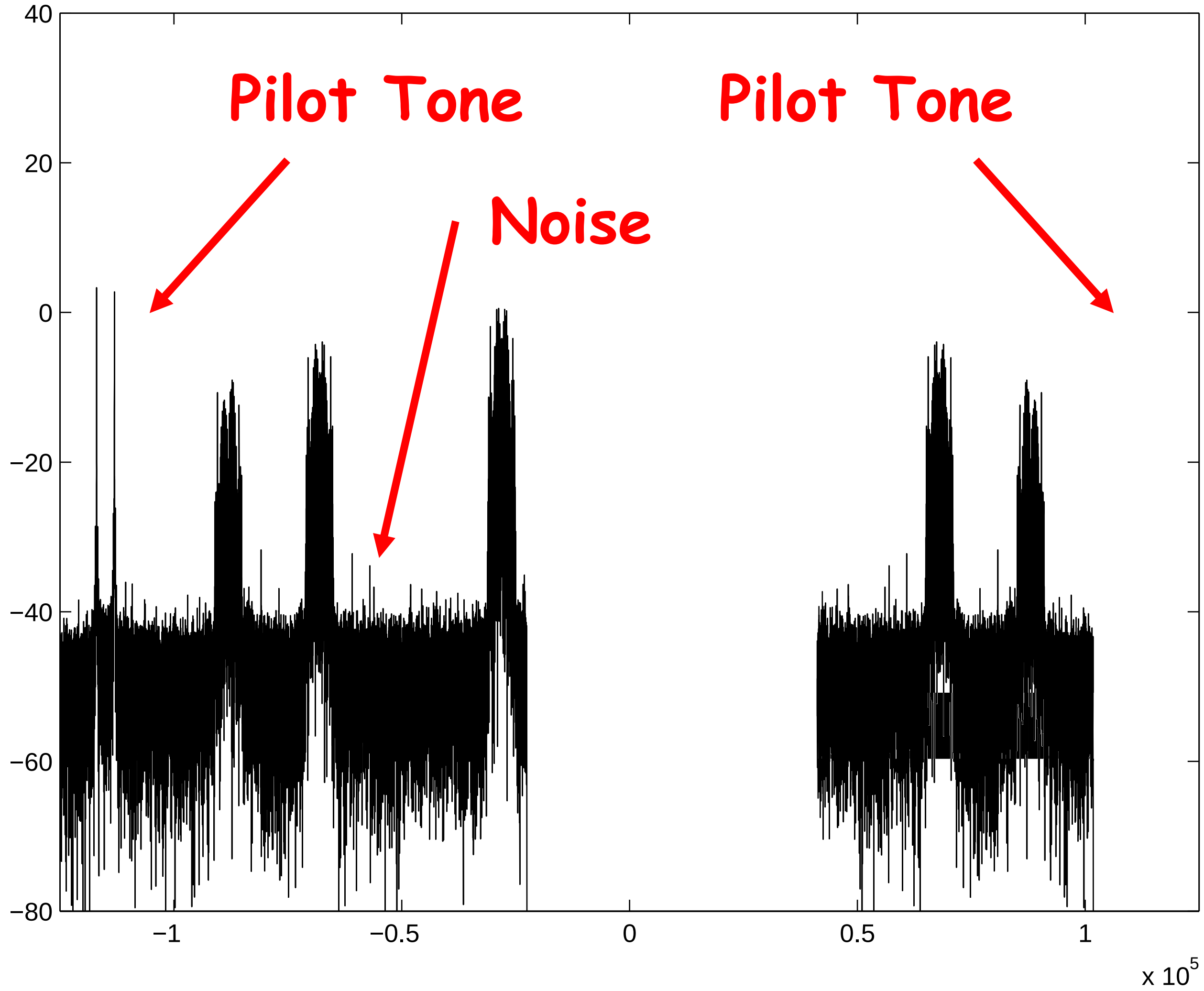


- Assuming we get our demodulation frequency and phase just right, we can *perfectly* reconstruct the originally transmitted signal at the receiver

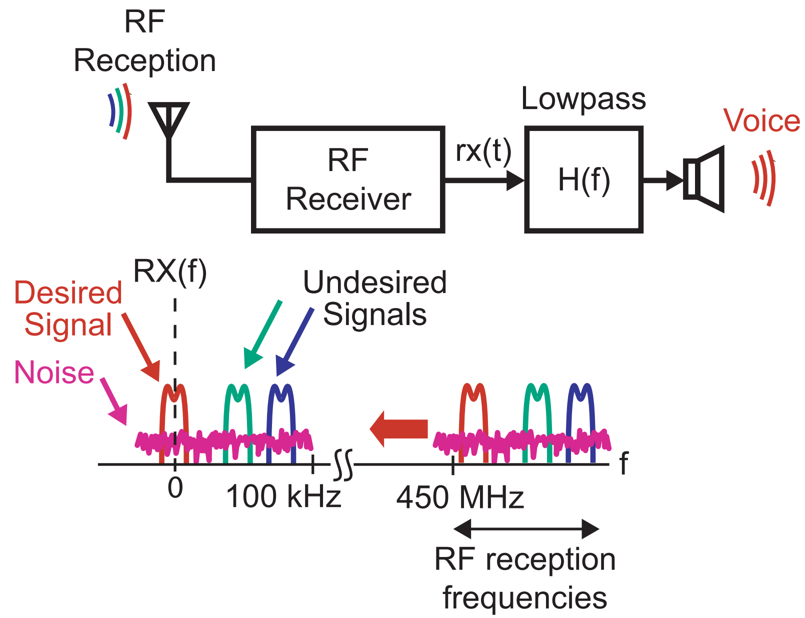
Is this true in the *real* world?

Lab 3 at a *Real* Lab Bench

- **Measured receive signal using monitor_receive('rx_a'):**

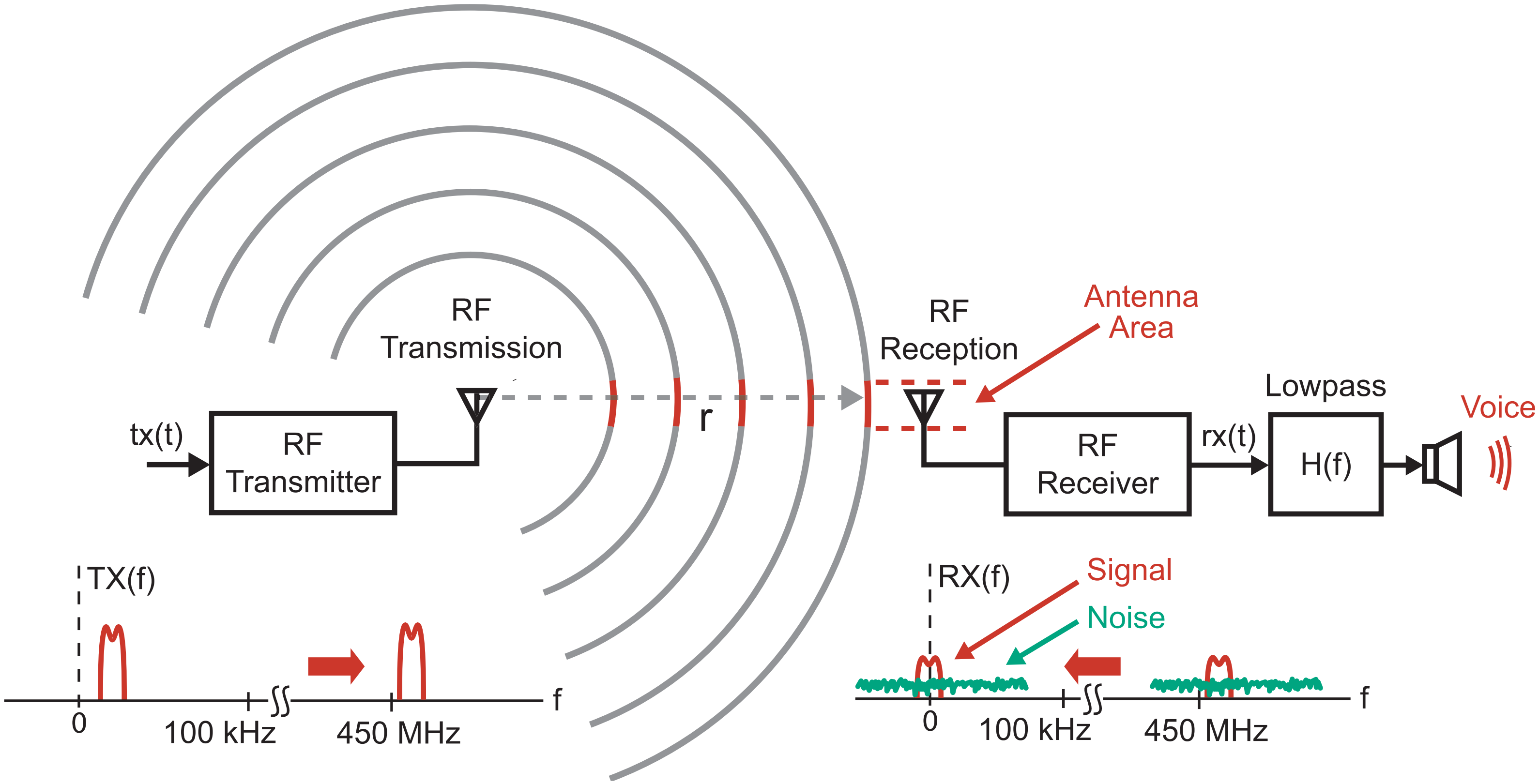


The Issue of Noise



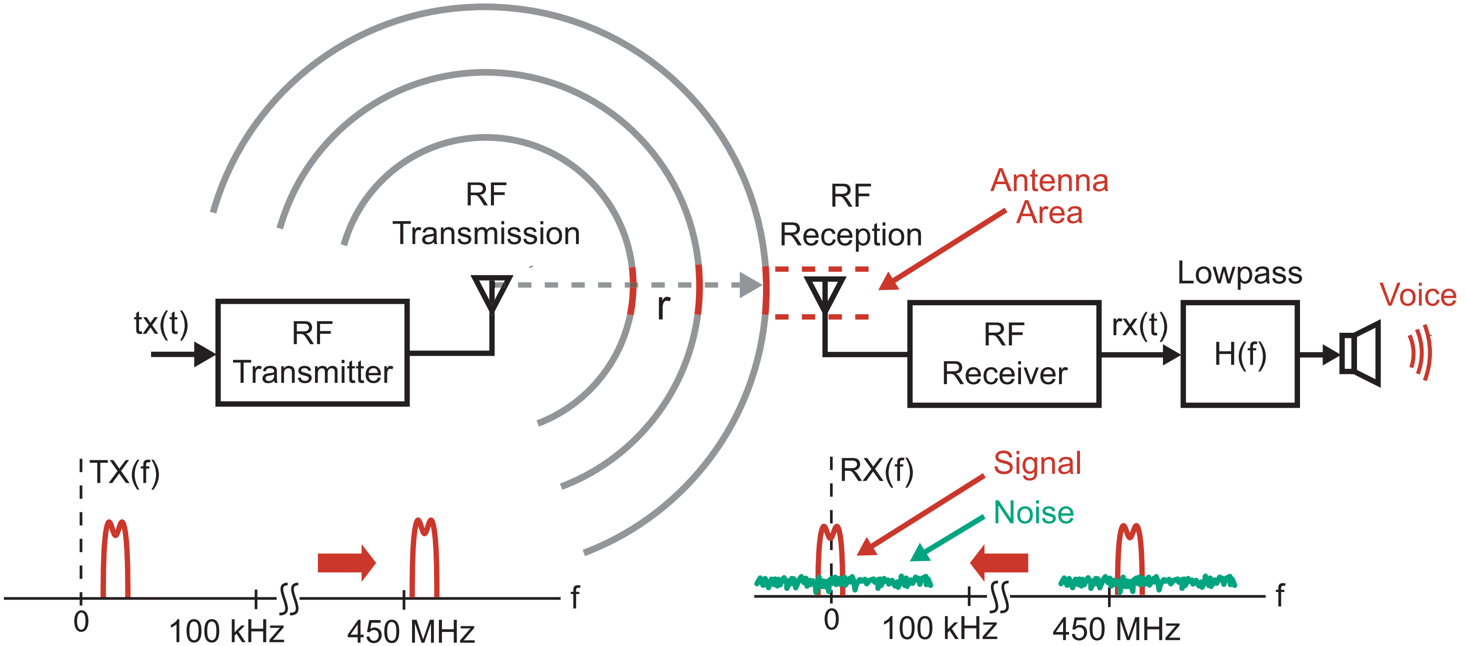
- Noise is a *non-predictable* (i.e. *random*), corrupting signal that adds to the desired signal
  - For RF receiver, most of it comes from the analog circuits that amplify and demodulate the input signal
- An undesired signal is a *predictable*, corrupting signal which also adds to the desired signal
  - May be called noise if it is difficult to predict

Energy Transfer in Wireless Communication



- Receiver antenna is limited in its ability to capture transmitter energy according to its *area* and *distance, r,* from transmitter
  - Received signal energy is a function of these parameters
  - In free space, received energy is proportional to $1/r^2$

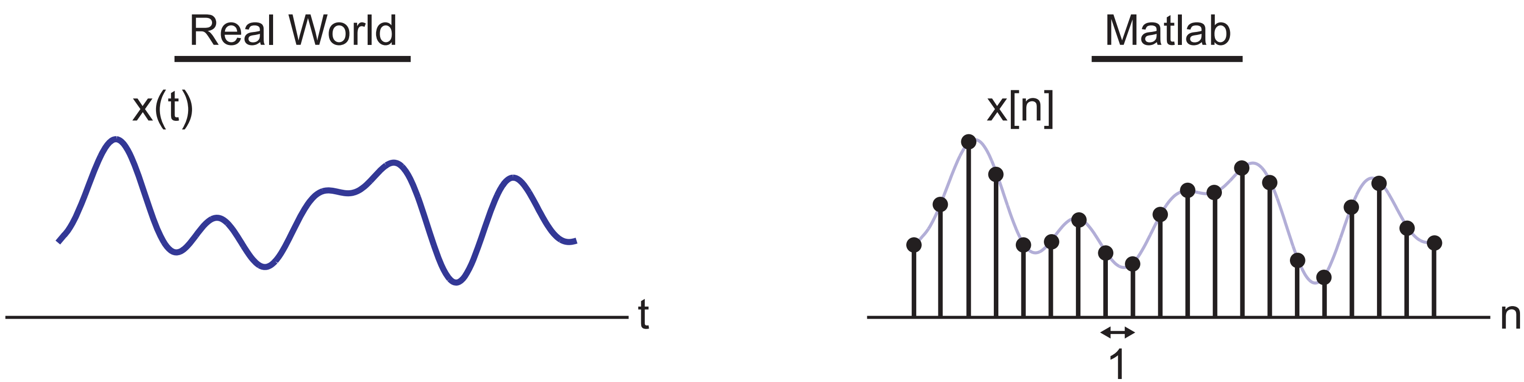
Signal Versus Noise



- **Moving the receiver closer to the transmitter *increases* desired *signal* energy**
  - ***Noise* from analog receiver circuits is *unchanged***

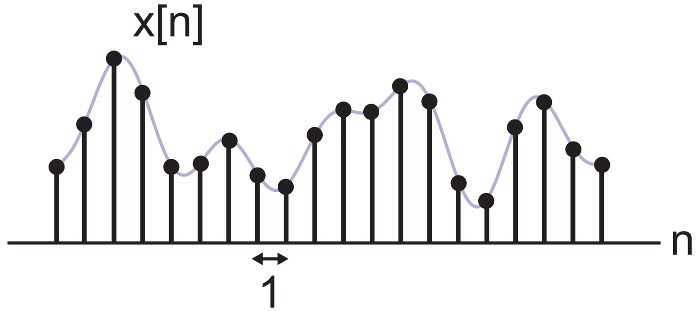
How is system performance impacted?

Development of *Metrics* for Analysis



- **It is often useful to create a mapping between a signal *waveform* and a *numerical value***
  - Such a mapping is called a *metric*
  - Examples: energy, power, average, variance
- **In this class, we prefer to do analysis on discrete-time signals**
  - Our labs focus on Matlab sequences rather than analog signals in the underlying hardware
  - Key ideas transfer to analog signal analysis quite readily

Definition of Mean, Power, and Energy



- **DC average or mean, μ_x, is defined as**

$$\mu_x = \frac{1}{N}\sum_{k=0}^{N-1} x[n]$$

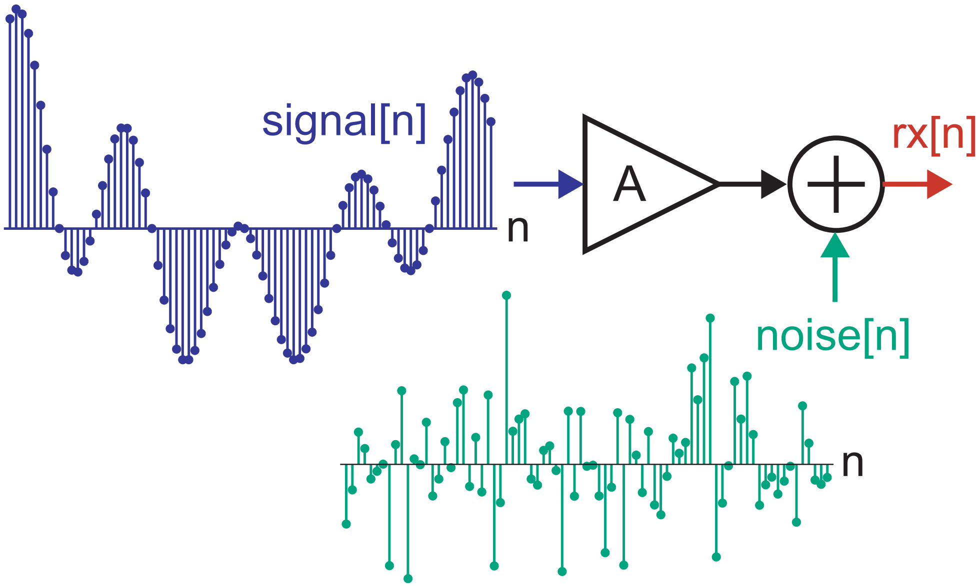
- **Power, P_x, and energy, E_x, are defined as**

$$P_x = \frac{1}{N}\sum_{k=0}^{N-1} x[n]^2 \qquad E_x = \sum_{k=0}^{N-1} x[n]^2$$

  - **For communication systems, we often remove the mean since it is essentially irrelevant in terms of *information*:**

$$\tilde{P}_x = \frac{1}{N}\sum_{k=0}^{N-1} (x[n]-\mu_x)^2 \qquad \tilde{E}_x = \sum_{k=0}^{N-1} (x[n]-\mu_x)^2$$

Definition of Signal-to-Noise Ratio



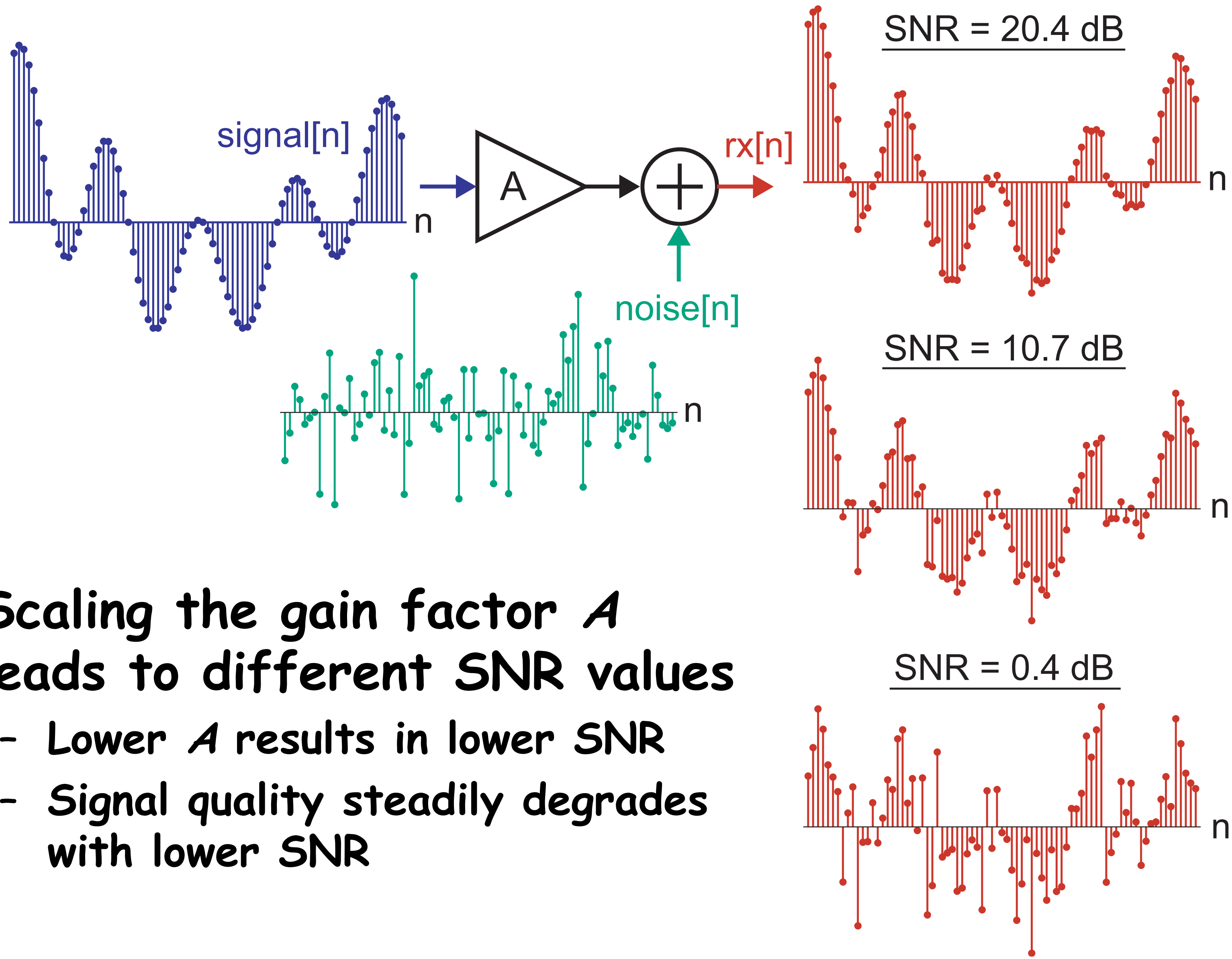
- **Signal-to-Noise ratio (SNR) indicates the relative impact of noise in system performance**

$$\mathrm{SNR} = \frac{\tilde{P}_{signal}}{\tilde{P}_{noise}}$$

- **We often like to use units of *dB* to express SNR:**

$$\mathrm{SNR}\ (\mathrm{dB}) = 10\log\left(\frac{\tilde{P}_{signal}}{\tilde{P}_{noise}}\right)$$

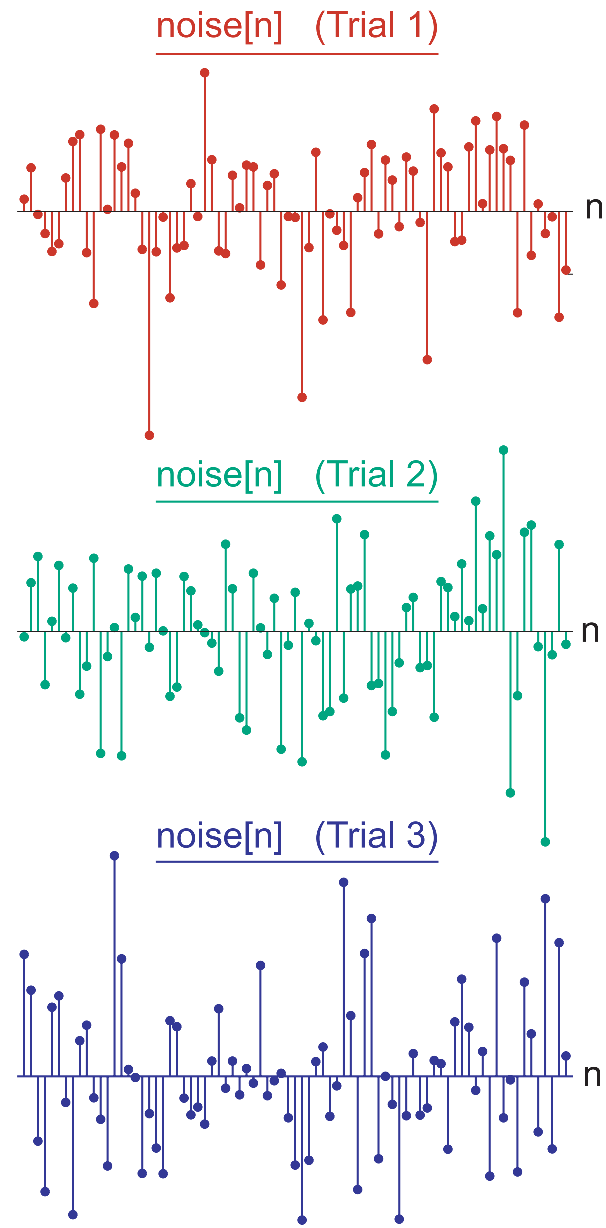
SNR Example



- **Scaling the gain factor *A* leads to different SNR values**
  - **Lower *A* results in lower SNR**
  - **Signal quality steadily degrades with lower SNR**

Analysis of *Random Processes*

- Random processes, such as noise, take on different sequences for different trials
  - Think of trials as different measurement intervals from the same experimental setup (as in Lab)
- For a *given* trial, we can apply our standard analysis tools and metrics
  - Fourier transform, mean and power calculations, etc…
- When trying to analyze the *ensemble* (i.e. *all* trials) of possible outcomes, we find ourselves in need of *new* tools and metrics

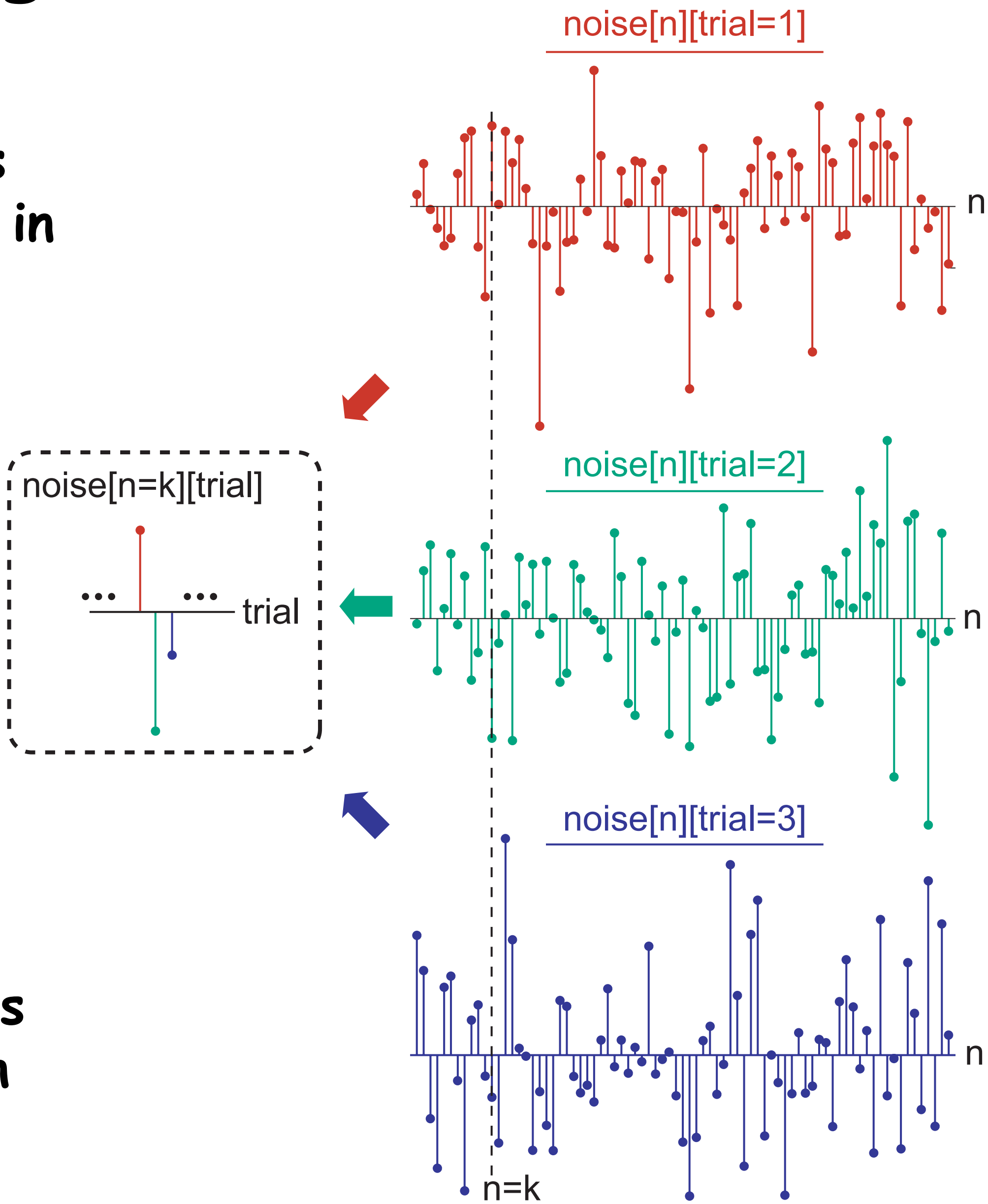


Tools and Metrics for Random Processes

- Assume that random processes we will deal with have the properties of being *stationary* and *ergodic*
  - True for noise in many practical communication systems
  - Greatly simplifies analysis – 6.011 will provide details
- Examine in both time and frequency domains
  - Time domain
    - Introduce the concept of a *probability density function* (PDF) to characterize behavior of signals at a given sample time
    - Use PDF to calculate mean and variance
      - Similar to mean and power of non-random signals
  - Frequency domain
    - We must wait for 6.011 to give you the proper framework
    - For now, we will simply use Fourier analysis on signals from *individual* trials as done in Labs
      - We will give some hints of *ensemble* behavior …

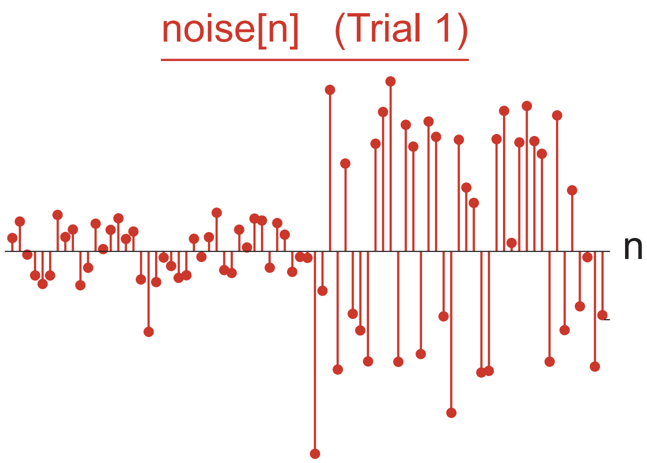
Stationary and Ergodic Random Processes

- **Stationary**
  - Statistical behavior is independent of *shifts* in *time* in a given trial:
    - Implies *noise[k]* is statistically indistinguishable from *noise[k+N]*
- **Ergodic**
  - Statistical *sampling* can be performed at one sample time (i.e., *n=k*) across *different* trials, *or* across different sample times of the *same* trial with no change in the measured result

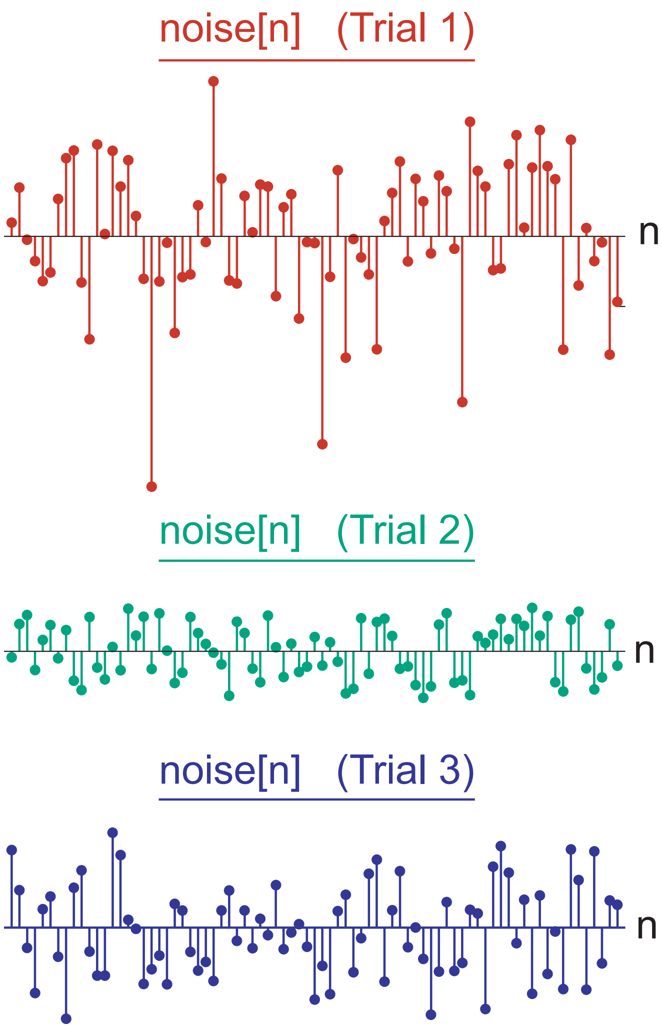


Examples

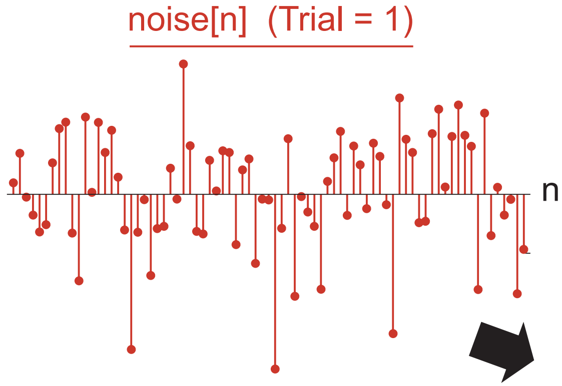
- **Non-Stationary**

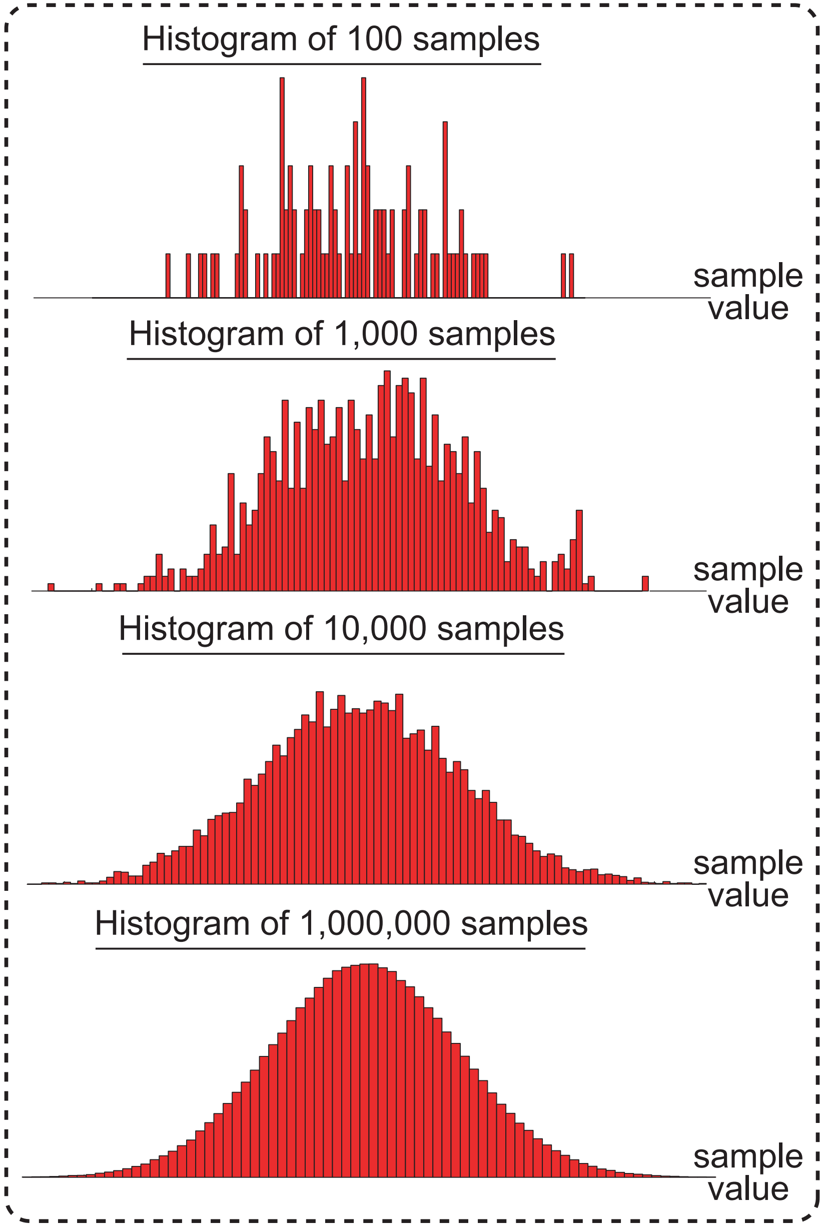


- **Stationary, but Non-Ergodic**



Experiment to see Statistical Distribution



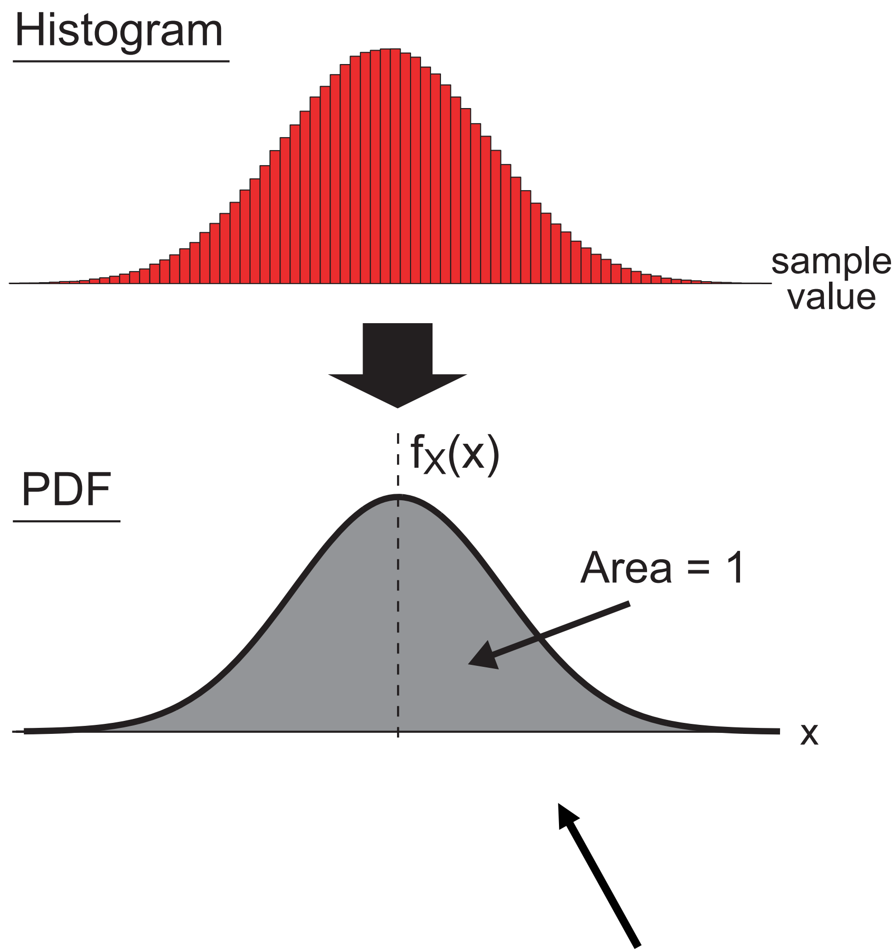


- Create histograms of sample values from trials of increasing lengths
- Assumption of stationarity implies histogram should converge to a shape known as a probability density function (PDF)

Formalizing the PDF Concept

- Define x as a random variable whose PDF has the same shape as the histogram we just obtained
- Denote PDF of x as $f_X(x)$
  - Scale $f_X(x)$ such that its overall area is 1

$$\Rightarrow \int_{-\infty}^{\infty} f_X(x) = 1$$

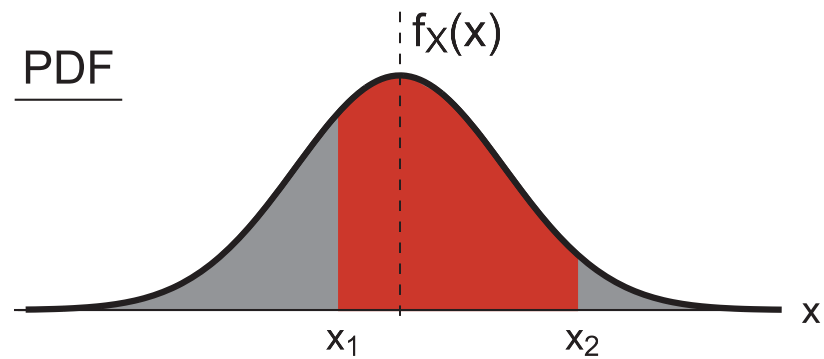


This shape is referred to as a *Gaussian* PDF

Formalizing Probability

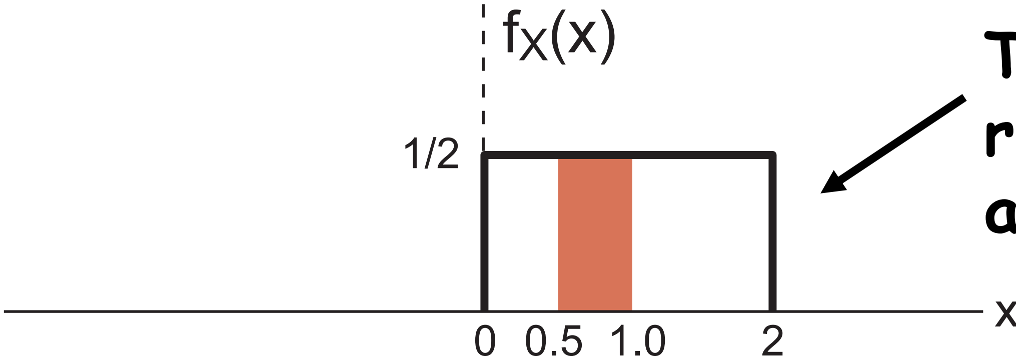
- The *probability* that random variable x takes on a value in the range of x_1 to x_2 is calculated from the PDF of x as:

$$\mathrm{Prob}(x_1 \le x \le x_2) = \int_{x_1}^{x_2} f_X(x)dx$$



- Note that probability values are always in the range of 0 to 1
  - Higher probability values imply greater likelihood that the event will occur

Example Probability Calculation



This shape is referred to as a *uniform* PDF

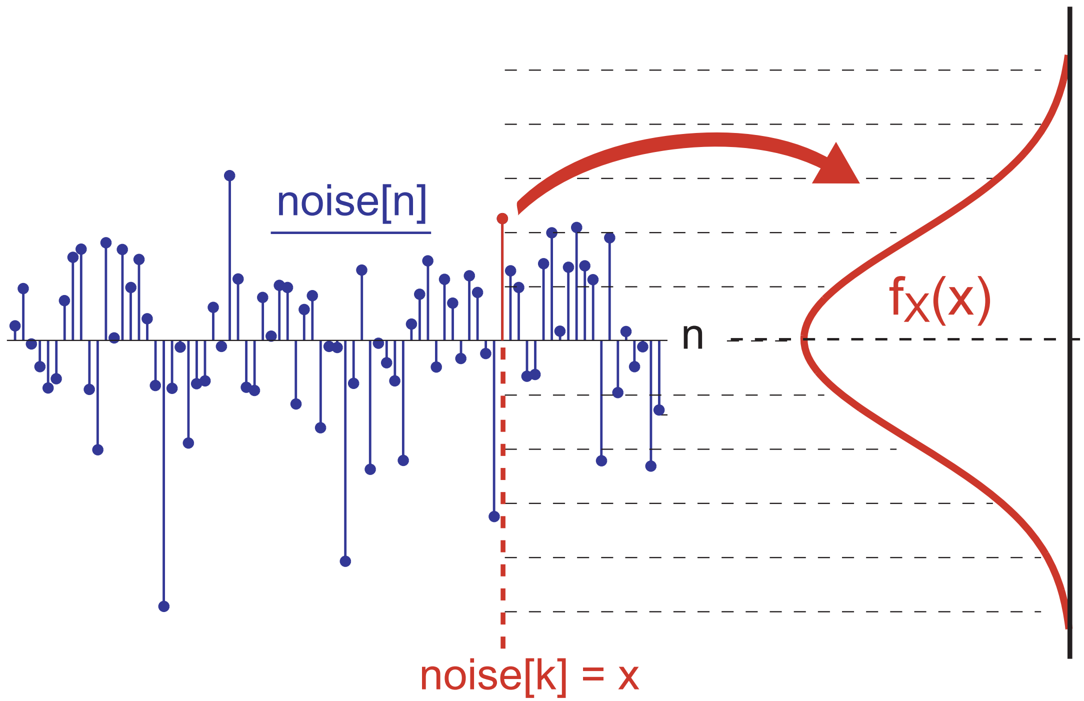
- Verify that overall area is 1:

$$\int_{-\infty}^{\infty} f_X(x)dx = \int_0^2 0.5\,dx \;=\; \boxed{1}$$

- Probability that x takes on a value between 0.5 and 1.0:

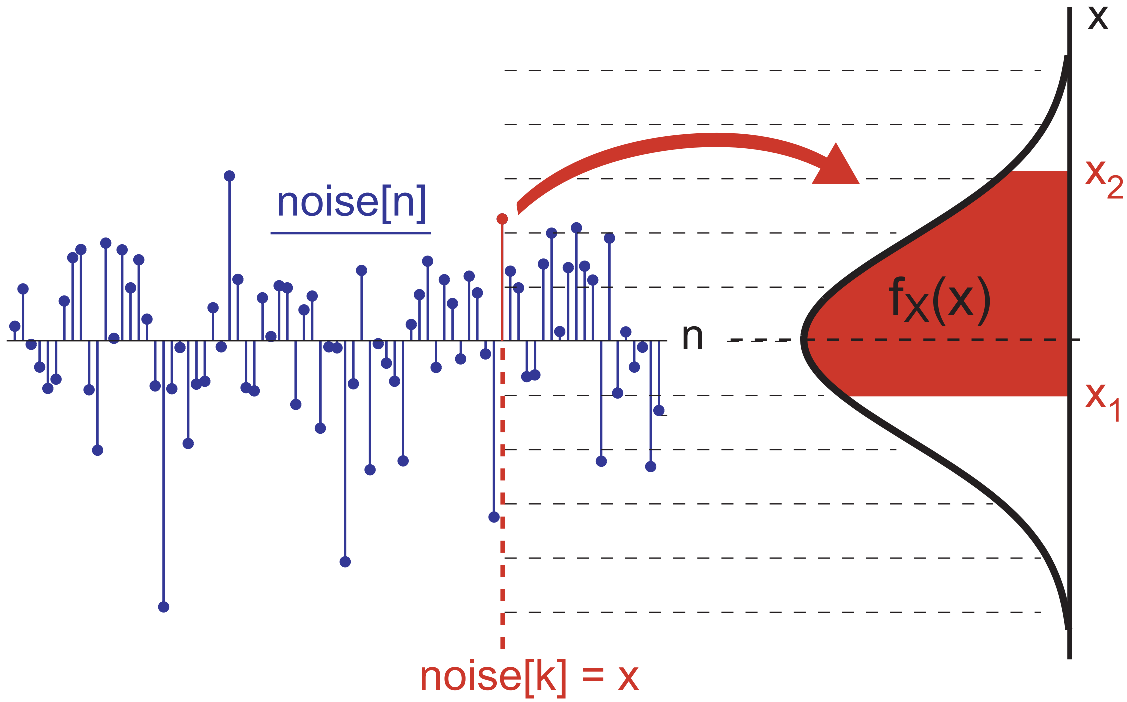
$$\mathrm{Prob}(0.5 \le x \le 1.0) = \int_{0.5}^{1.0} 0.5\,dx \;=\; \boxed{0.25}$$

Examination of Sample Value Distribution



- Assumption of ergodicity implies the value occurring at a *given* time sample, *noise[k]*, across *many different* trials has the *same PDF* as estimated in our previous experiment of *many* time samples and *one* trial
- We can model *noise[k]* as the random variable *x*

Probability Calculation

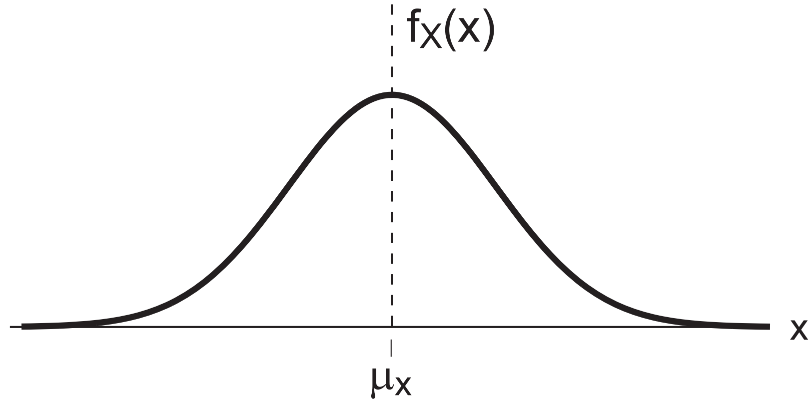


- In a given trial, the *probability* that *noise[k]* takes on a value in the range of x_1 to x_2 is computed as

$$\mathrm{Prob}(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f_X(x)dx$$

We will return to this when we analyze performance of digital modulation systems

Mean and Variance



- **The mean of random variable *x*, μ_x, corresponds to its average value**
  - **Computed as**

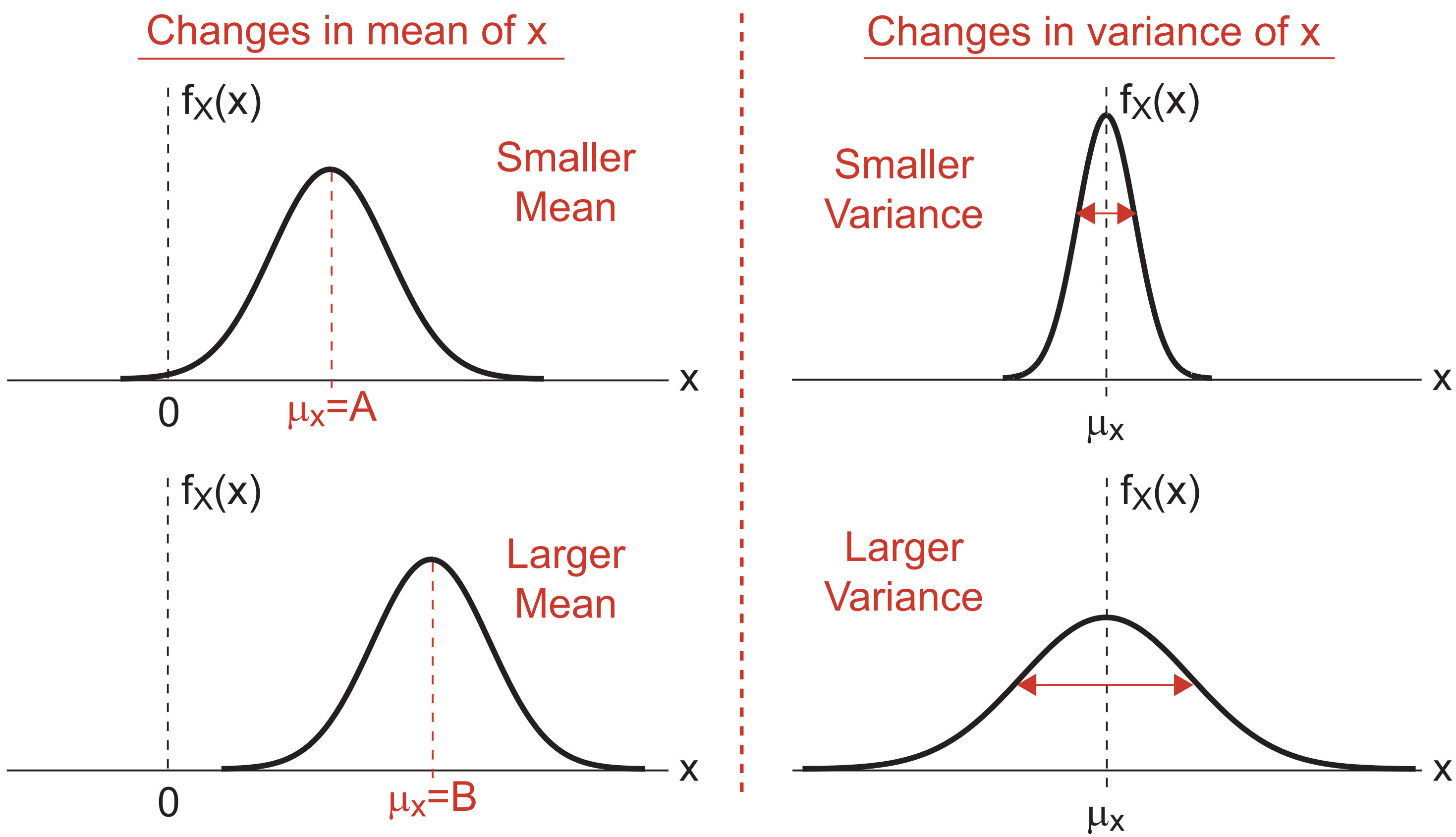
$$\mu_x = \int_{-\infty}^{\infty} x f_X(x) dx$$

- **The variance of random variable *x*, σ^2_x, gives an indication of its variability**
  - **Computed as**

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) dx$$

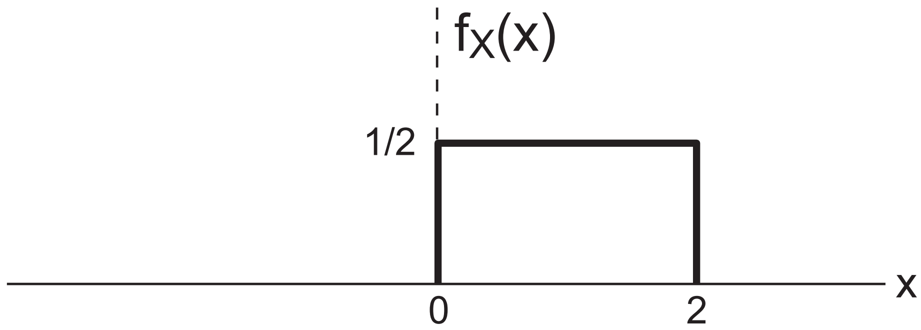
  - **Similar to power of a signal**

Visualizing Mean and Variance from PDF



- Changes in mean shift the *center of mass* of PDF
- Changes in variance narrow or broaden the PDF
  - Note that area of PDF must always remain equal to one

Example Mean and Variance Calculation



- **Mean:**

$$\mu_x = \int_{-\infty}^{\infty} x f_X(x) dx = \int_0^2 x \frac{1}{2} dx = \frac{1}{4} x^2 \Big|_0^2 = \boxed{1}$$

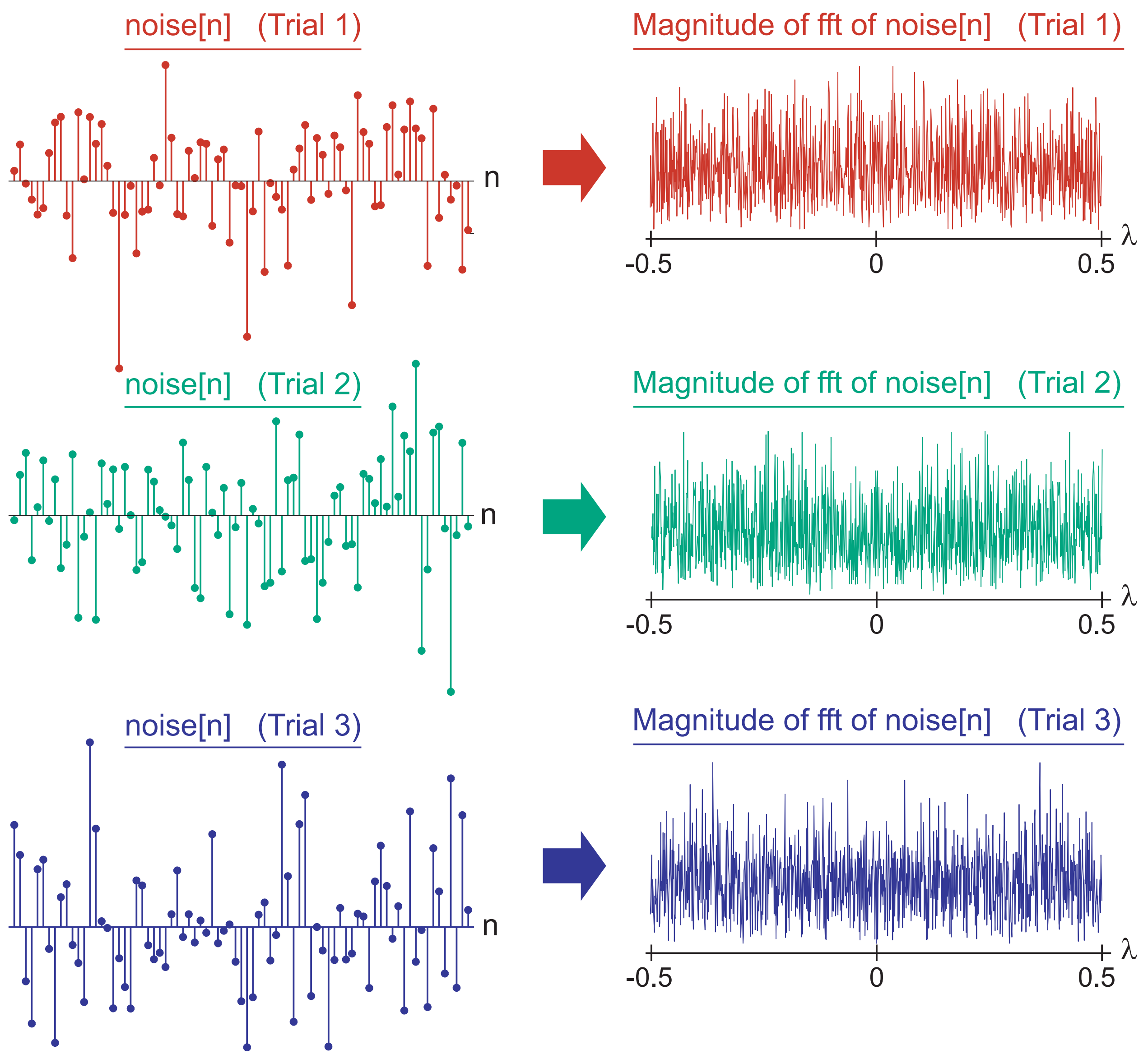
- **Variance:**

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) dx = \int_0^2 (x-1)^2 \frac{1}{2} dx$$

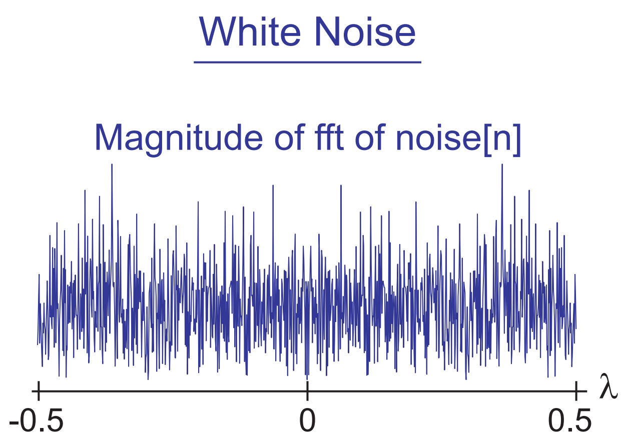
$$= \frac{1}{6}(x-1)^3 \Big|_0^2 = \frac{1}{6} + \frac{1}{6} = \boxed{\frac{1}{3}}$$

Frequency Domain View of Random Process

- It is valid to take *fft* of sequence from a given trial
  - We did this in lab
- Notice that the *fft* result changes for different trials
  - We saw this in the *spectrum* plots of lab 3
  - Fourier Transform is undefined !

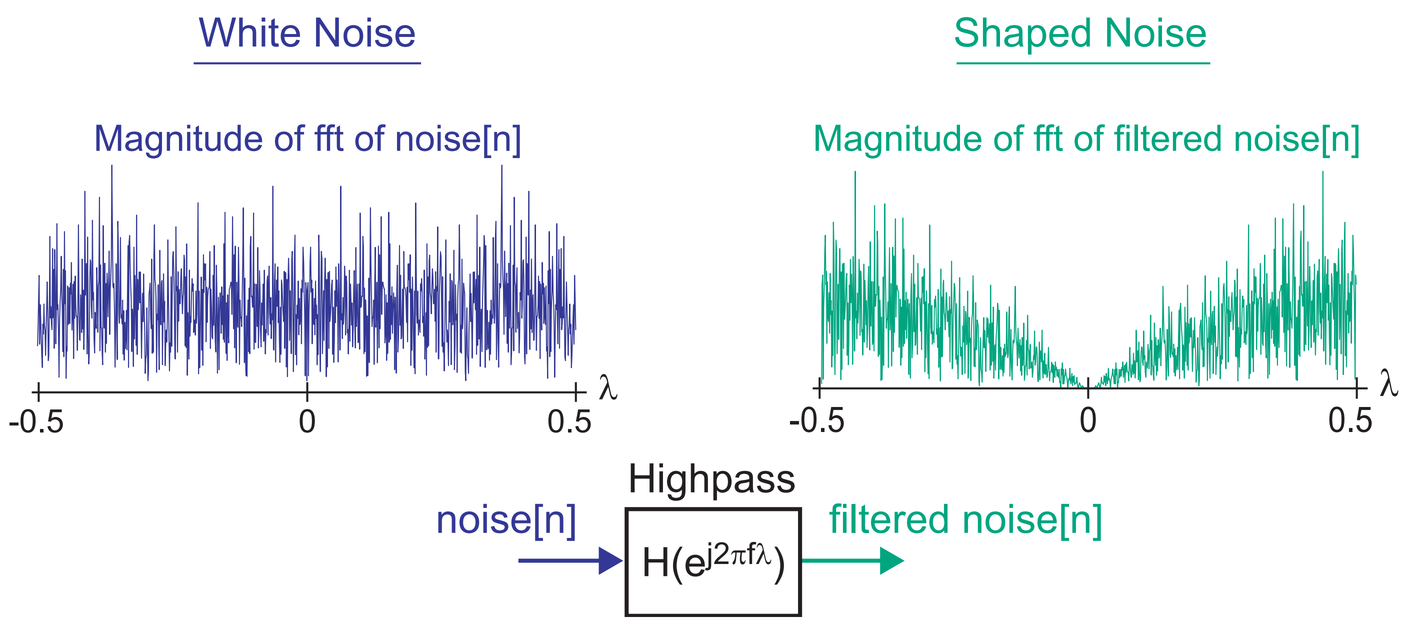


White Noise



- When the *fft* result (i.e. *spectrum*) looks relatively flat, we refer to the random process as being white
  - Note: this type of noise source is often used for calibration of advanced stereo systems

Shaped Noise



- **Shaped noise occurs when white noise is sent into a filter**
  - ***fft* of shaped noise will have frequency content according to the type of filter**
    - **Example: highpass filter yields shaped noise with only high frequency content**

Summary

- **A useful metric characterizing the performance of communication systems is Signal-to-Noise Ratio**
  - High SNR values are desirable
  - SNR often varies in a wireless system according to the distance between transmitter and receiver

- **Analysis of random processes (such as noise) requires additional tools**
  - Concepts of stationarity and ergodicity
  - Random variables and their associated PDF functions
  - Metrics such as mean and variance

- **Take 6.011 to learn more about random processes**
  - Proper framework for frequency domain analysis
  - Advanced topics such as estimation