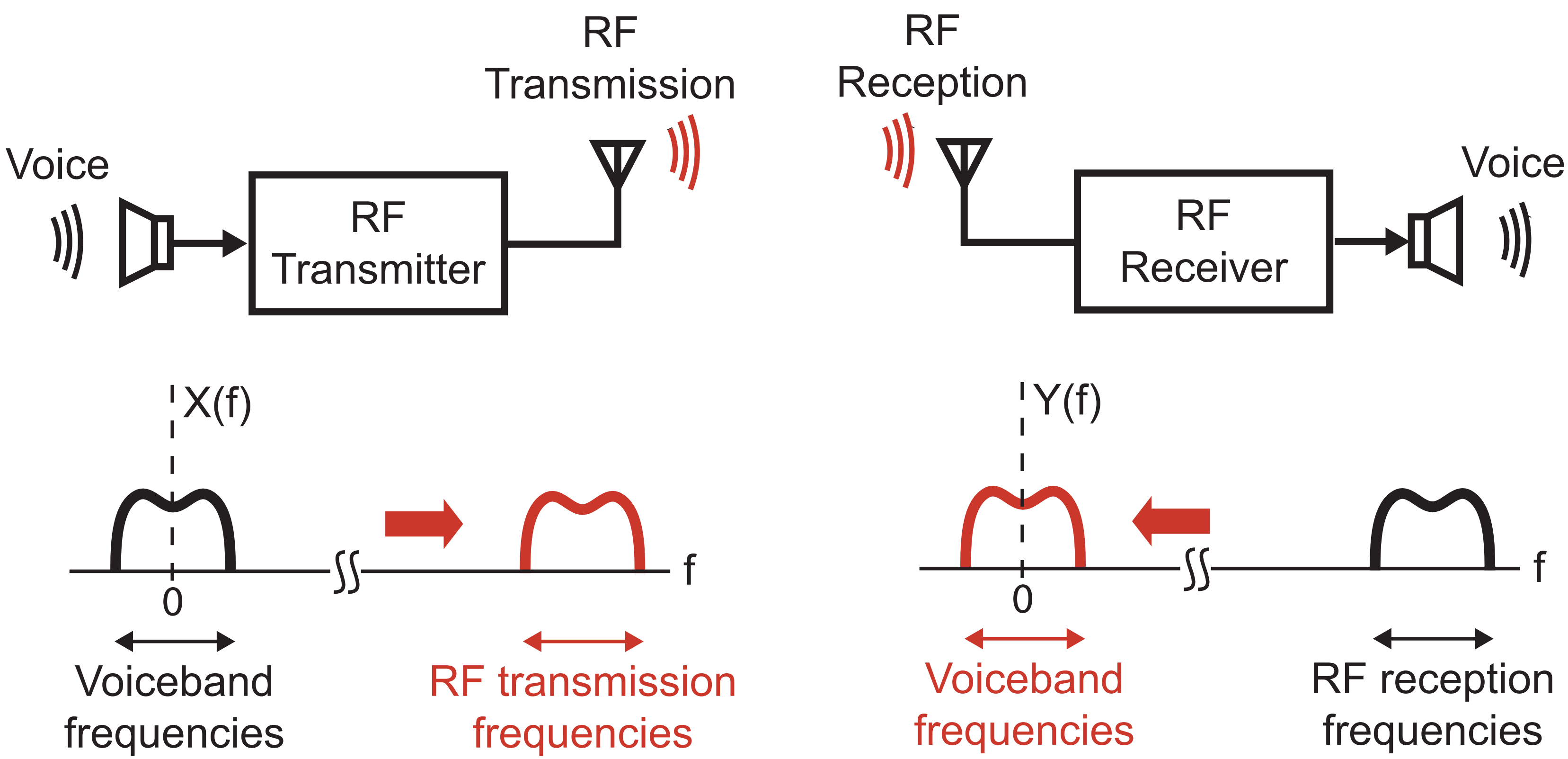
Modulation and Filtering

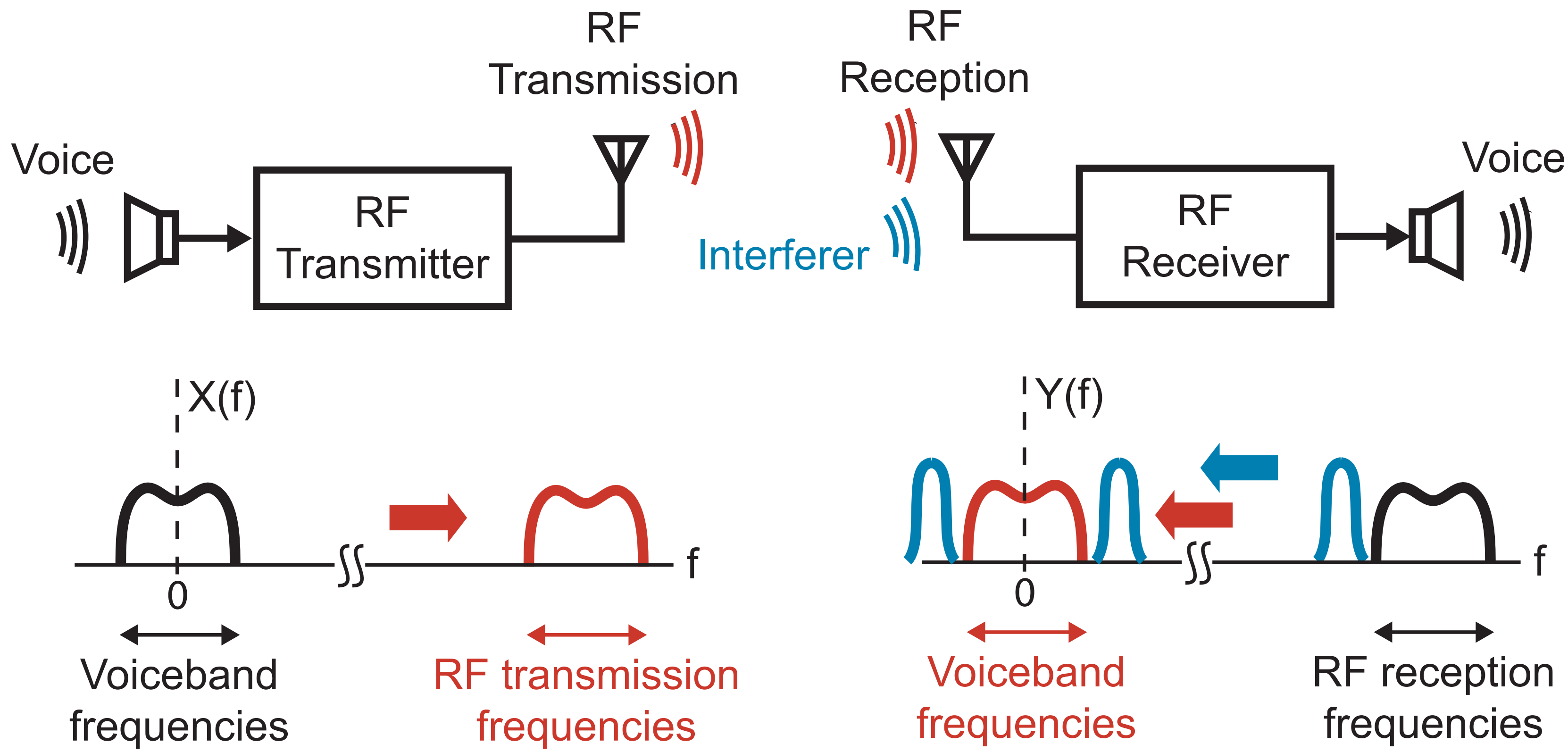
- Wireless communication application
- Impulse function definition and properties
- Fourier Transform of Impulse, Sine, Cosine
- Picture analysis using Fourier Transforms

Motivation for Modulation



- **Modulation is used to change the frequency band of a signal**
  - **Enables RF communication in different frequency bands**
    - **Used in cell phones, AM/FM radio, WLAN, cable TV, ….**
  - **Note: higher frequencies lead to smaller antennas**

Motivation for Filtering



- **Filtering is used to remove undesired signals outside of the frequency band of interest**
  - **Enables selection of a specific radio, TV, WLAN, cell phone, cable TV *channel* …**
  - **Undesired channels are often called interferers**

The Fourier Transform as a Tool

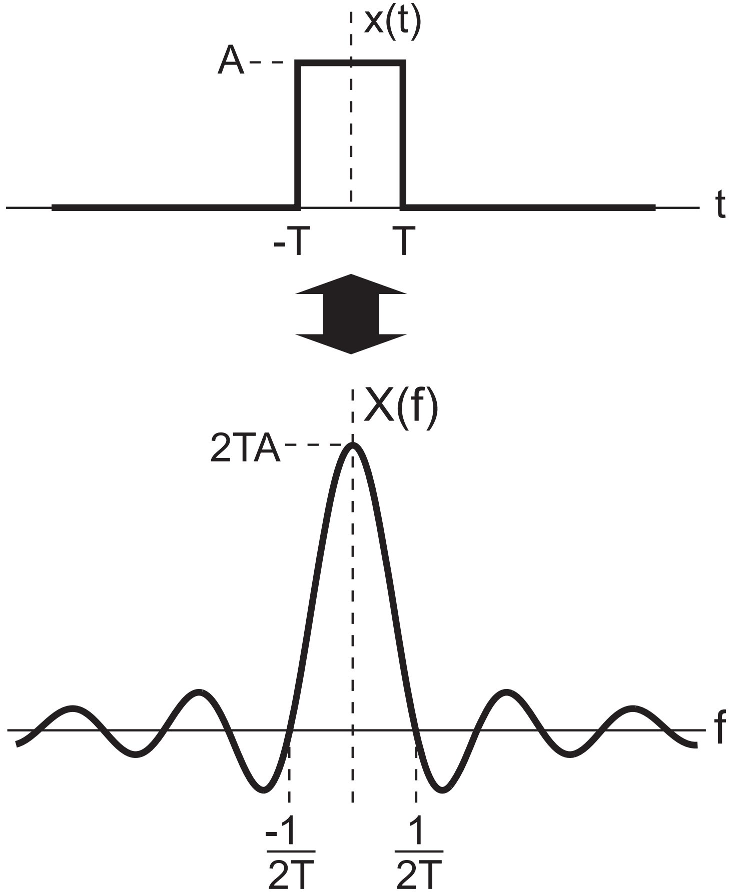
- Communication signals are often *non-periodic*
- Fourier Transforms allow us to do modulation and filtering analysis using *pictures*

$$x(t) \Leftrightarrow X(f)$$

Where:

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

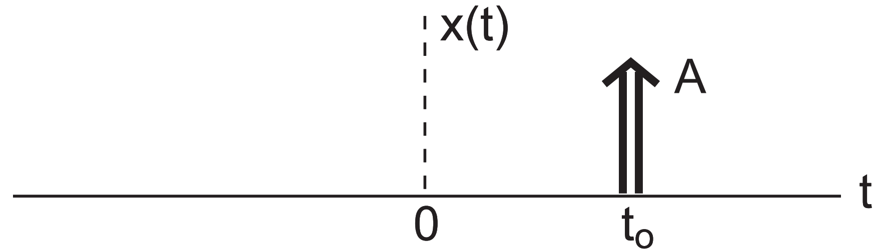
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$



Definition of the Impulse Function

- **An impulse of area *A* at time t_o is denoted as:**

$$A\delta(t - t_o)$$



- **Impulses are defined in terms of their properties**
  - **Area:**

$$\int_{-\infty}^{\infty} A\delta(t - t_o) = A$$

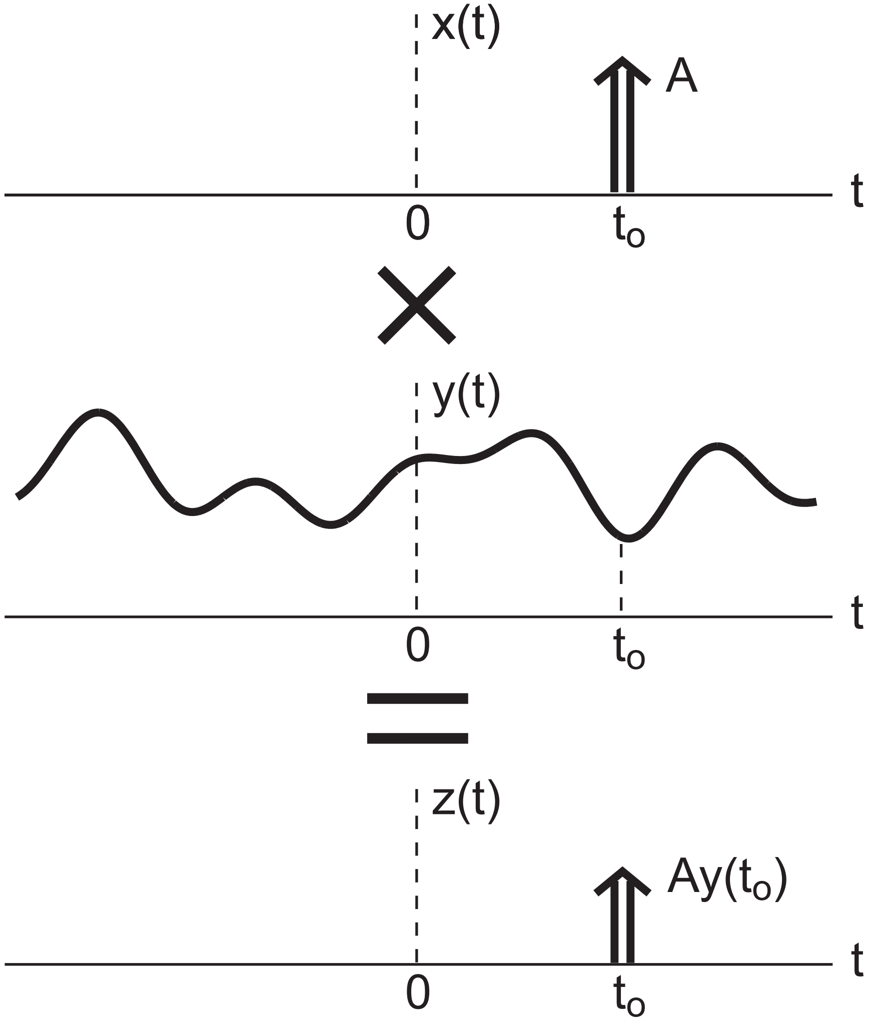
  - **Fourier Transform:**

$$A\delta(t - t_o) \Leftrightarrow Ae^{-j2\pi f t_o}$$

  - **Sampling and convolution properties**
    - **Shown on the next two slides**

Sampling Property of Impulses

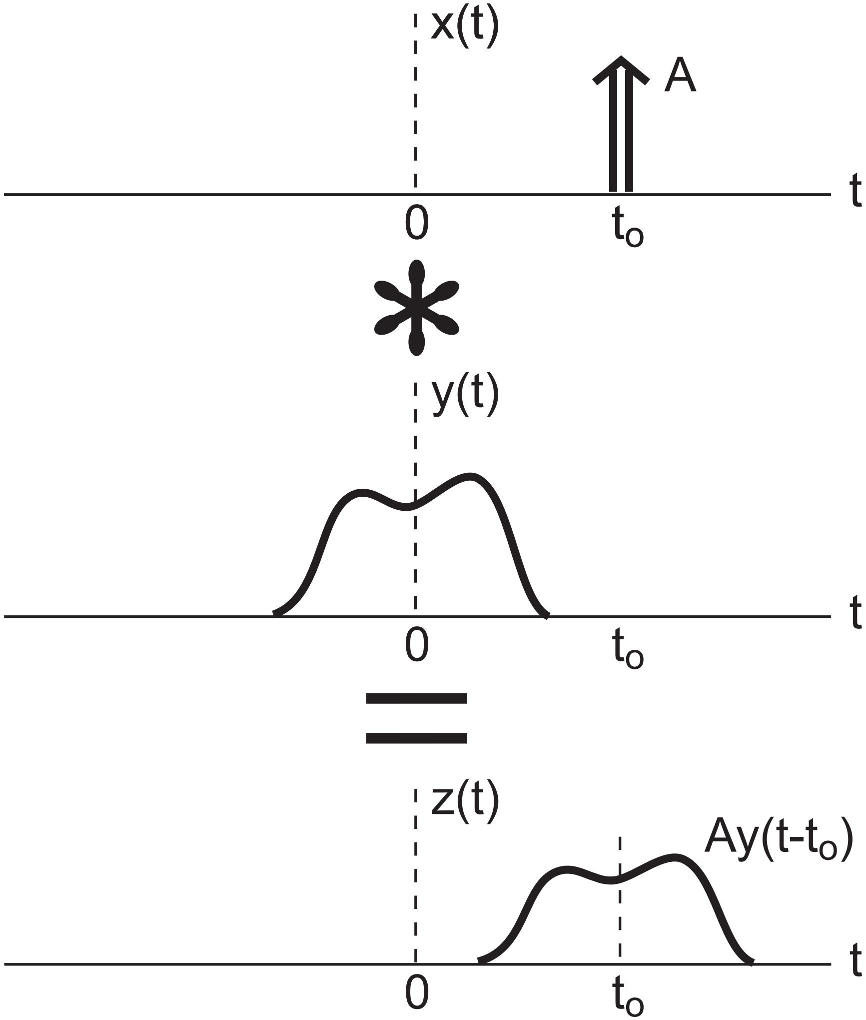
- *Multiplication* of an impulse and a continuous function leads to *scaling* of the *original impulse*
  - The scale factor corresponds to the *sample value* of the continuous function at the impulse location



$$A\delta(t - t_o)y(t) = Ay(t_o)\delta(t - t_o)$$

Convolution Property of Impulses

- *Convolution* of an impulse and a function leads to *shifting* and *scaling* of the *original function*
  - The shift value corresponds to the location of the impulse
  - The scale factor corresponds to the area of the impulse
- Convolution is not limited to impulses
  - 6.003 will explore this in great detail



$$A\delta(t-t_o) * y(t) = Ay(t-t_o)$$

Duality of Multiplication And Convolution

- Multiplication in time leads to convolution in frequency:

$$x(t)y(t) \;\Leftrightarrow\; X(f) * Y(f)$$

  - This is a key property to understand modulation
- Convolution in time leads to multiplication in frequency:

$$x(t) * y(t) \;\Leftrightarrow\; X(f)Y(f)$$

  - This is a key property to understand filtering
    - We will defer to 6.003 to give you more details here
  - We will use this fact in a few weeks to intuitively show the connection between the Fourier Series and Fourier Transform

Fourier Transform of Cosine Wave

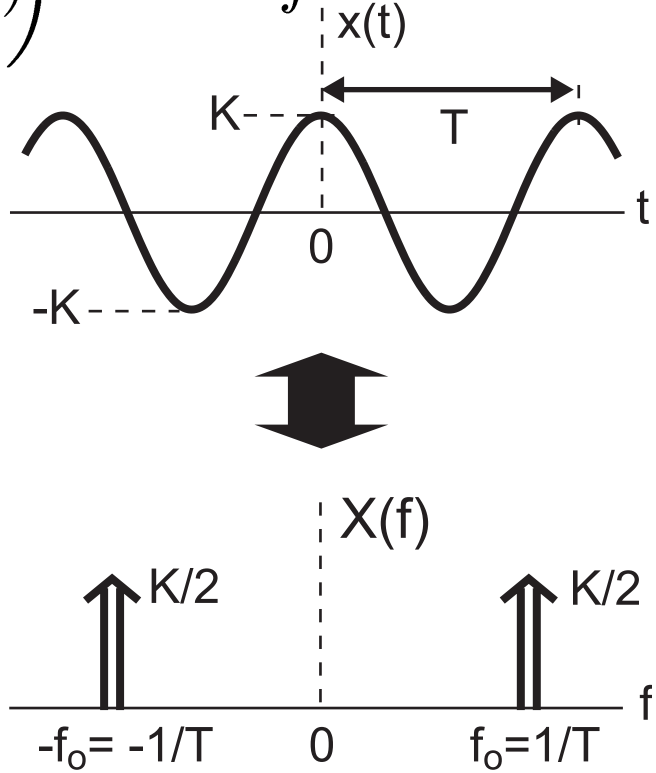
- **Two real impulses in frequency needed for cosine in time**

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

$$= \int_{-\infty}^{\infty} \frac{K}{2}\left(\delta(f+f_o) + \delta(f-f_o)\right) e^{j2\pi f t} df$$

$$= \frac{K}{2}\left(e^{-j2\pi f_o t} + e^{j2\pi f_o t} \right)$$

$$= K\cos(2\pi f_o t)$$



$$K\cos(2\pi f_o t) \Updownarrow \frac{K}{2}\left(\delta(f+f_o) + \delta(f-f_o)\right)$$

Fourier Transform of Sine Wave

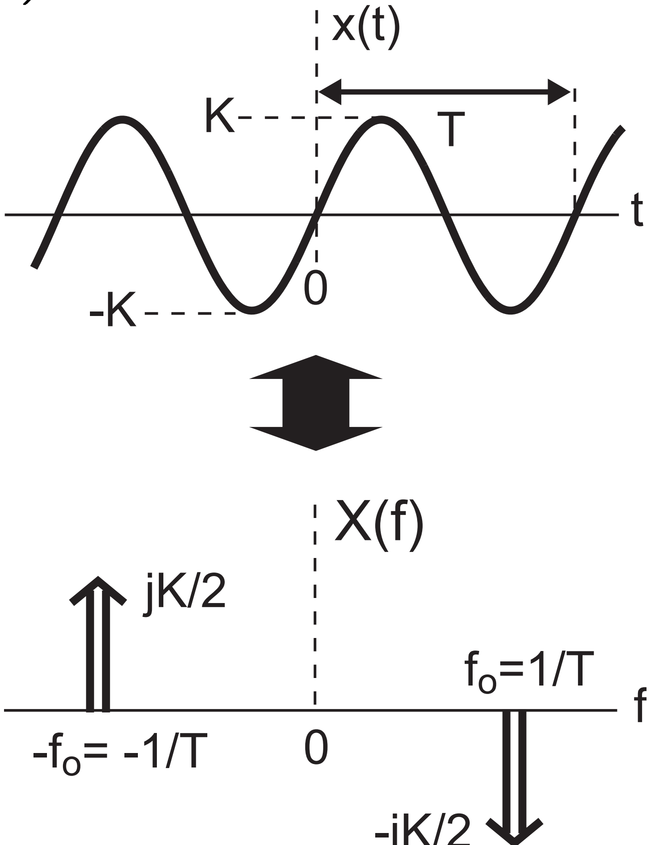
- **Two imaginary impulses in frequency needed for sine in time**

$$x(t) = \int_{-\infty}^{\infty} \frac{jK}{2}\Big(\delta(f+f_o) - \delta(f-f_o)\Big)\, e^{j2\pi ft} df$$

$$= \frac{jK}{2}\left(e^{-j2\pi f_o t} - e^{j2\pi f_o t}\right)$$

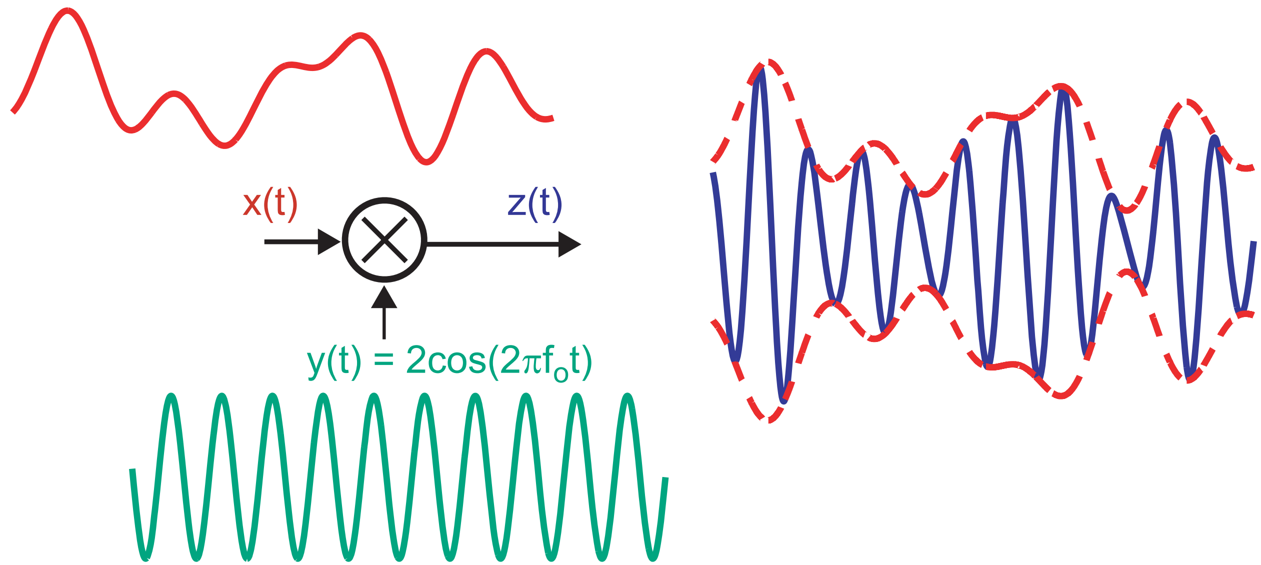
$$= \frac{K}{j2}\left(-\, e^{-j2\pi f_o t} + e^{j2\pi f_o t}\right)$$

$$= K\sin(2\pi f_o t)$$



$$K\sin(2\pi f_o t) \Longleftrightarrow j\frac{K}{2}\big(\delta(f+f_o) - \delta(f-f_o)\big)$$

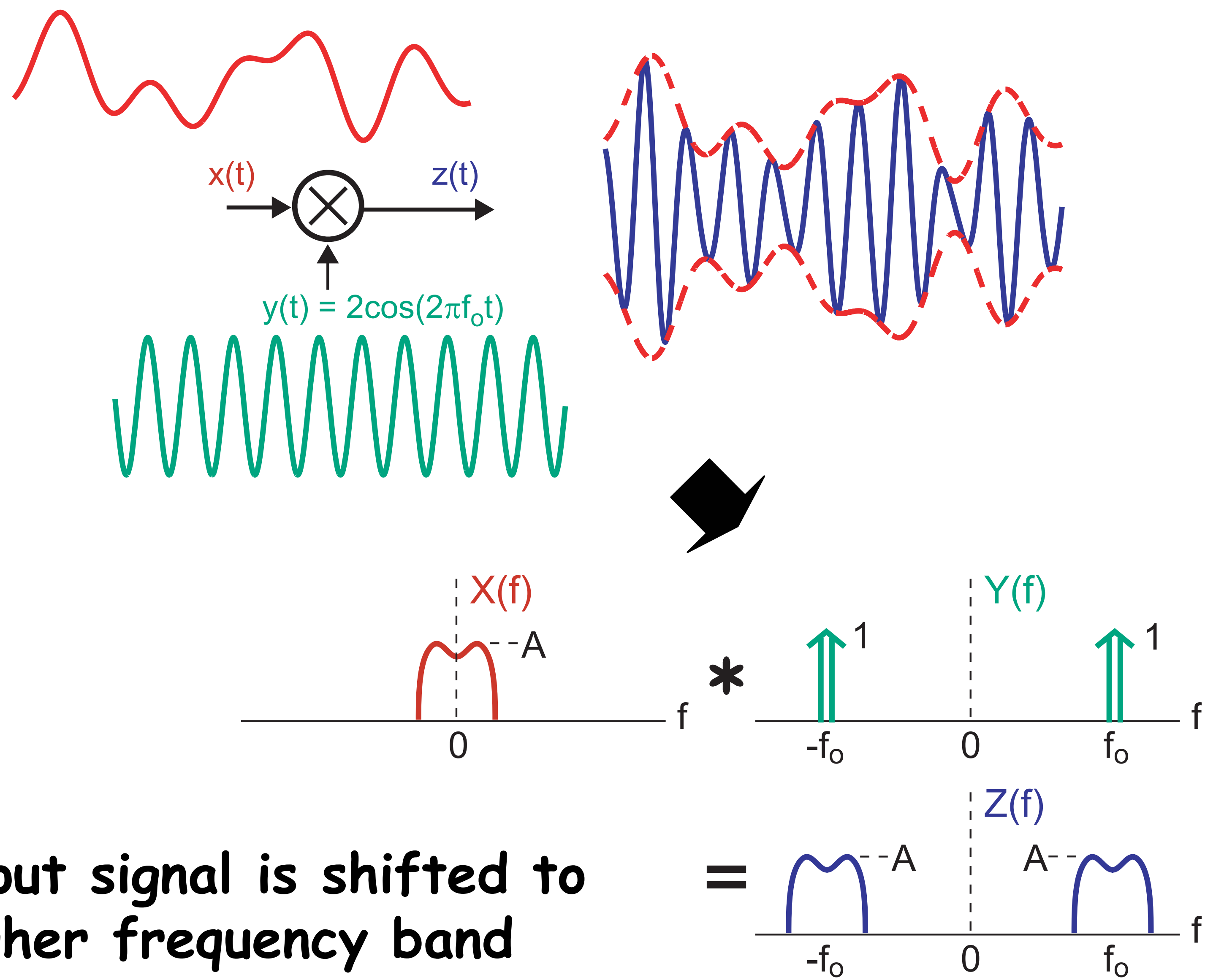
AM Modulation (Transmitter)



- **AM stands for *amplitude* modulation**
  - Frequency and phase modulation are also commonly used
- **Key operation is to *multiply* (i.e. *mix*) an input signal with a cosine (or sine) wave**
  - This leads to an oscillating waveform whose amplitude varies according to the input signal
- **Analysis:**

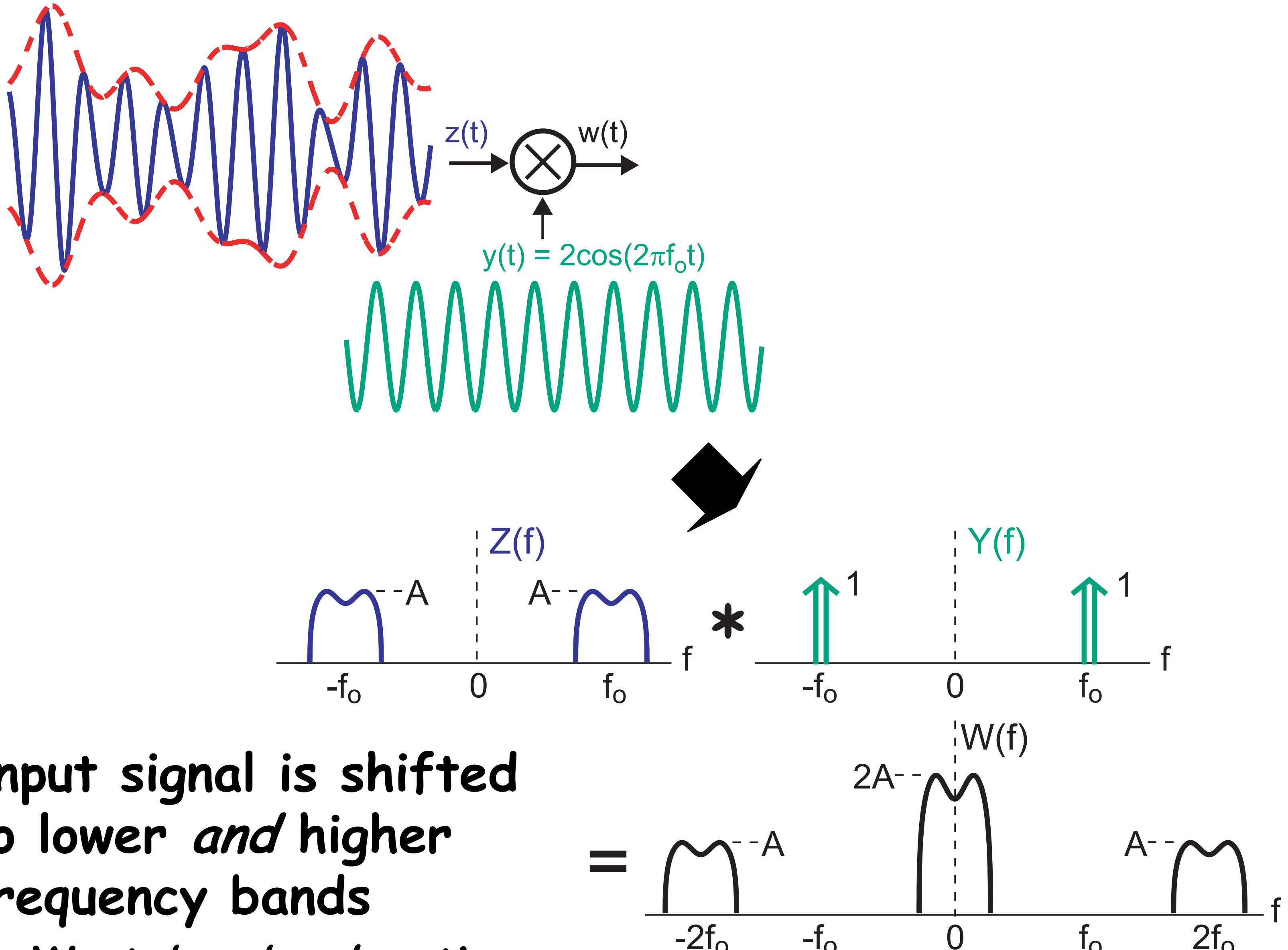
$$x(t)y(t) \Leftrightarrow X(f) * Y(f)$$

Fourier Transform Allows *Picture* Analysis



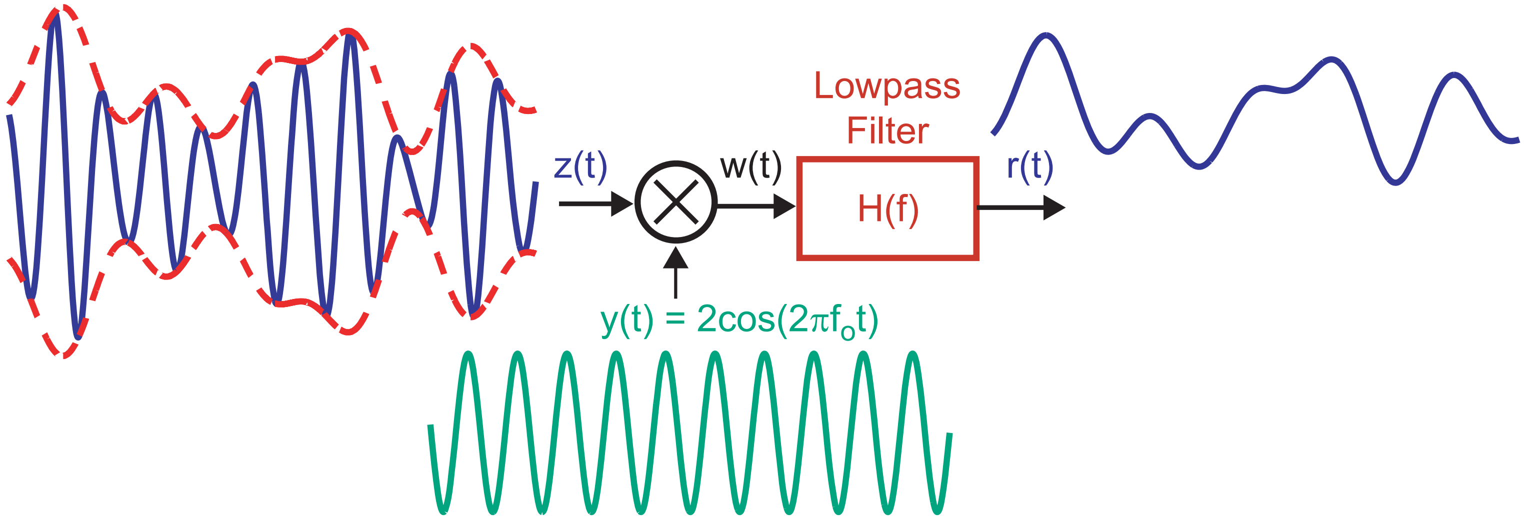
- **Input signal is shifted to higher frequency band**

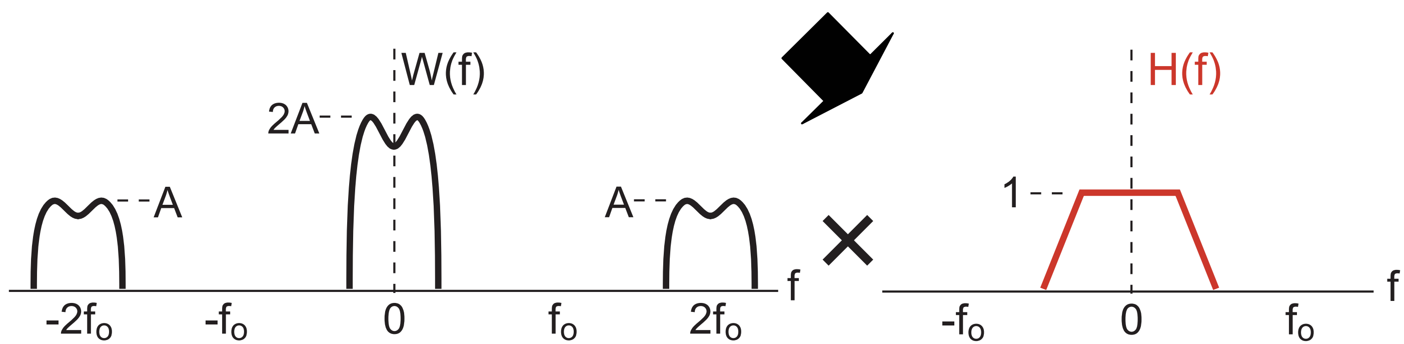
AM Demodulation (Receiver)

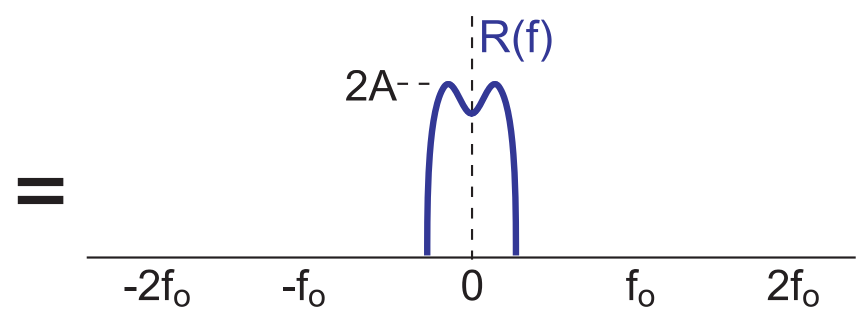


- **Input signal is shifted to lower *and* higher frequency bands**
  - **Want *baseband* portion**

Apply Lowpass Filter

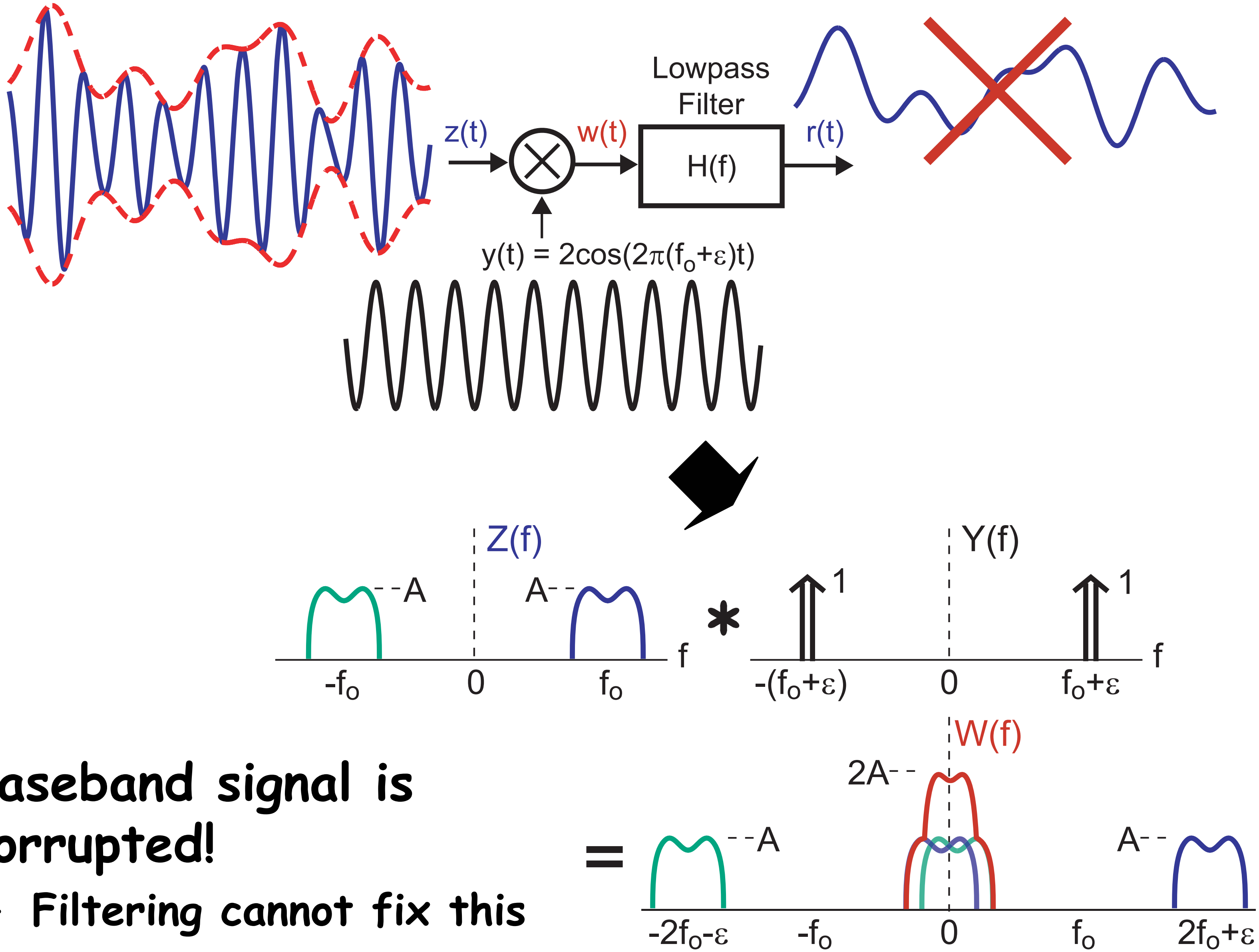






- **High frequency band portion is eliminated with lowpass filter**
  - **Only *baseband* portion remains**

Impact of Frequency Offset



- **Baseband signal is corrupted!**
  - **Filtering cannot fix this**

Summary

- The impulse function is an important concept for Fourier Transform analysis
  - Fourier Transforms of cosines and sines consist of impulses
  - Defined in terms of its properties
    - Area, Multiplication (sampling), Convolution
- The Fourier Transform allows picture analysis of modulation and filtering
  - Modulation *shifts* in frequency (convolution with impulses)
  - Filtering *multiplies* in frequency
- More details on filtering in next lecture
  - Design of filters in Matlab (for Lab exercises)
    - This tool only works with *discrete-time* signals
  - Discrete-Time Fourier Transform introduced