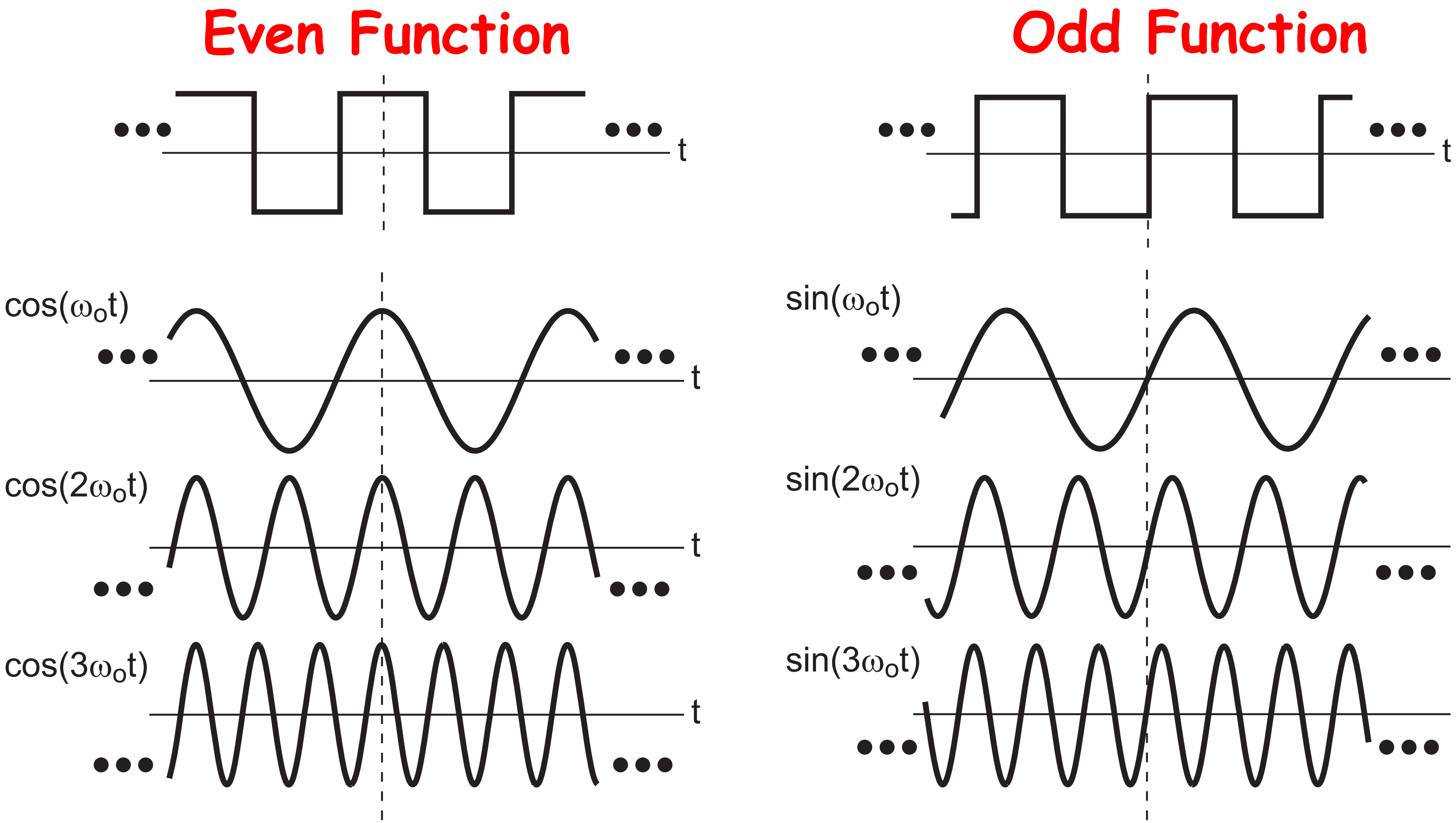
Wrap Up

- Fourier Transform
- Sampling, Modulation, Filtering
- Noise and the Digital Abstraction
- Binary signaling model and Shannon Capacity

Cosines and Sines as Basis Functions

- Periodic functions can be approximated by the addition of weighted cosine and sine waveforms with progressively increasing frequency



Fourier Series and Fourier Transform

- **The Fourier Series deals with *periodic* signals**

$$x(t) = \sum_{n=-\infty}^{\infty} \hat{X}_n e^{jnw_o t}$$

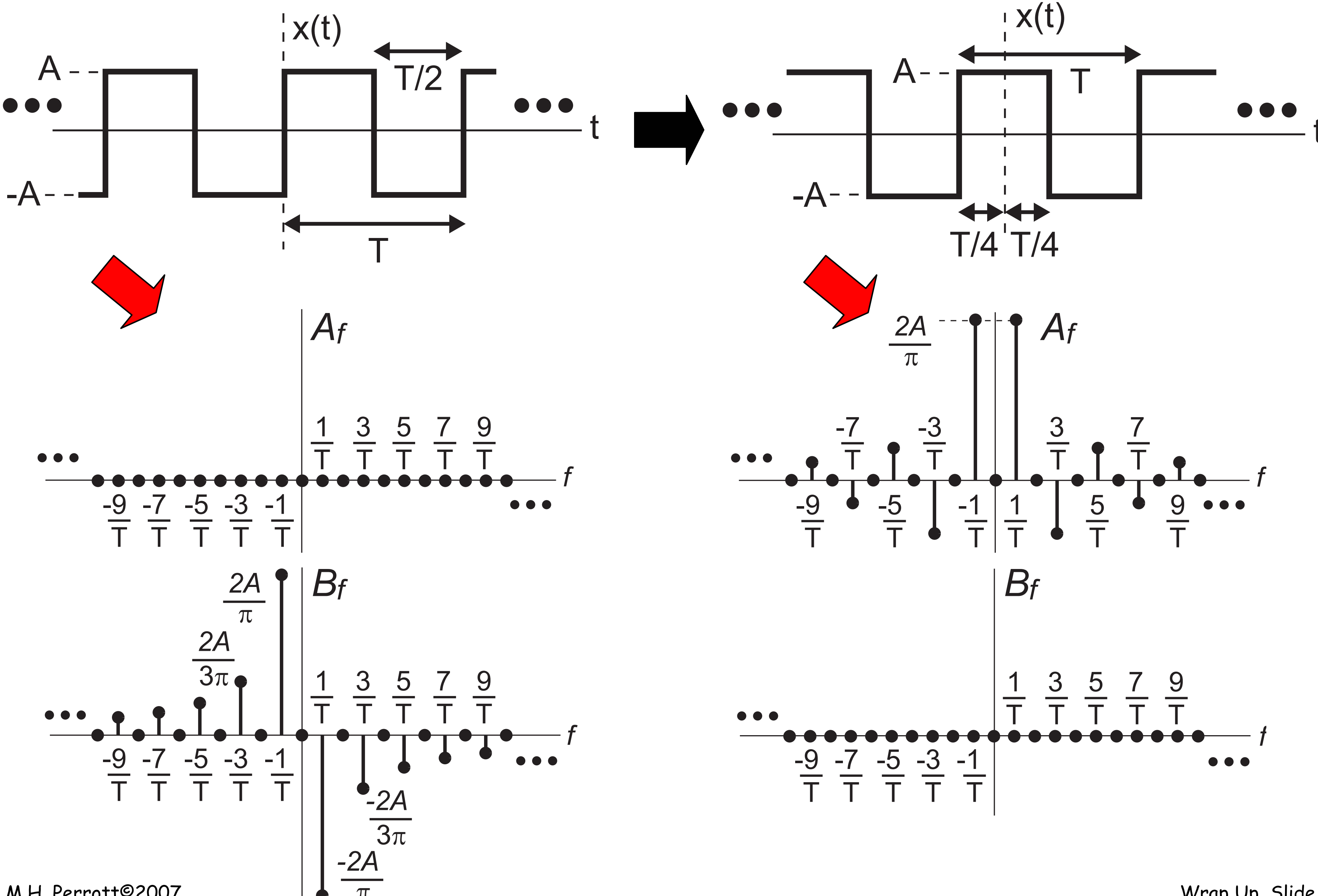
$$\hat{X}_n = \frac{1}{T}\int_{t_o}^{t_o+T} x(t)e^{-jnw_o t}dt$$

- **The Fourier Transform deals with *non-periodic* signals**

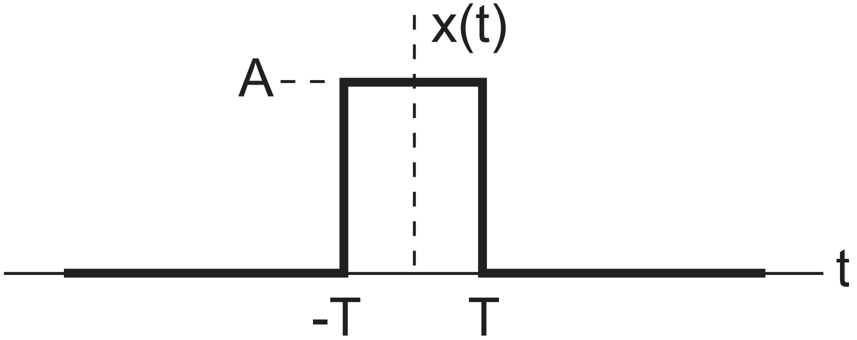
$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t}df$$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}dt$$

Graphical View of Fourier Series

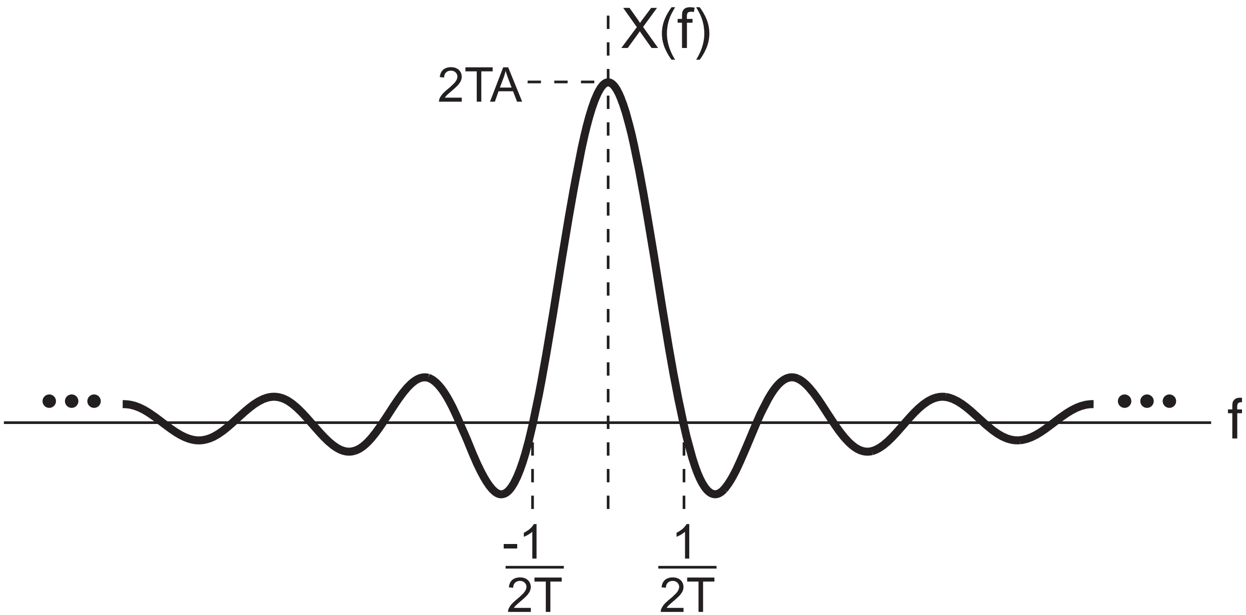


Graphical View of Fourier Transform

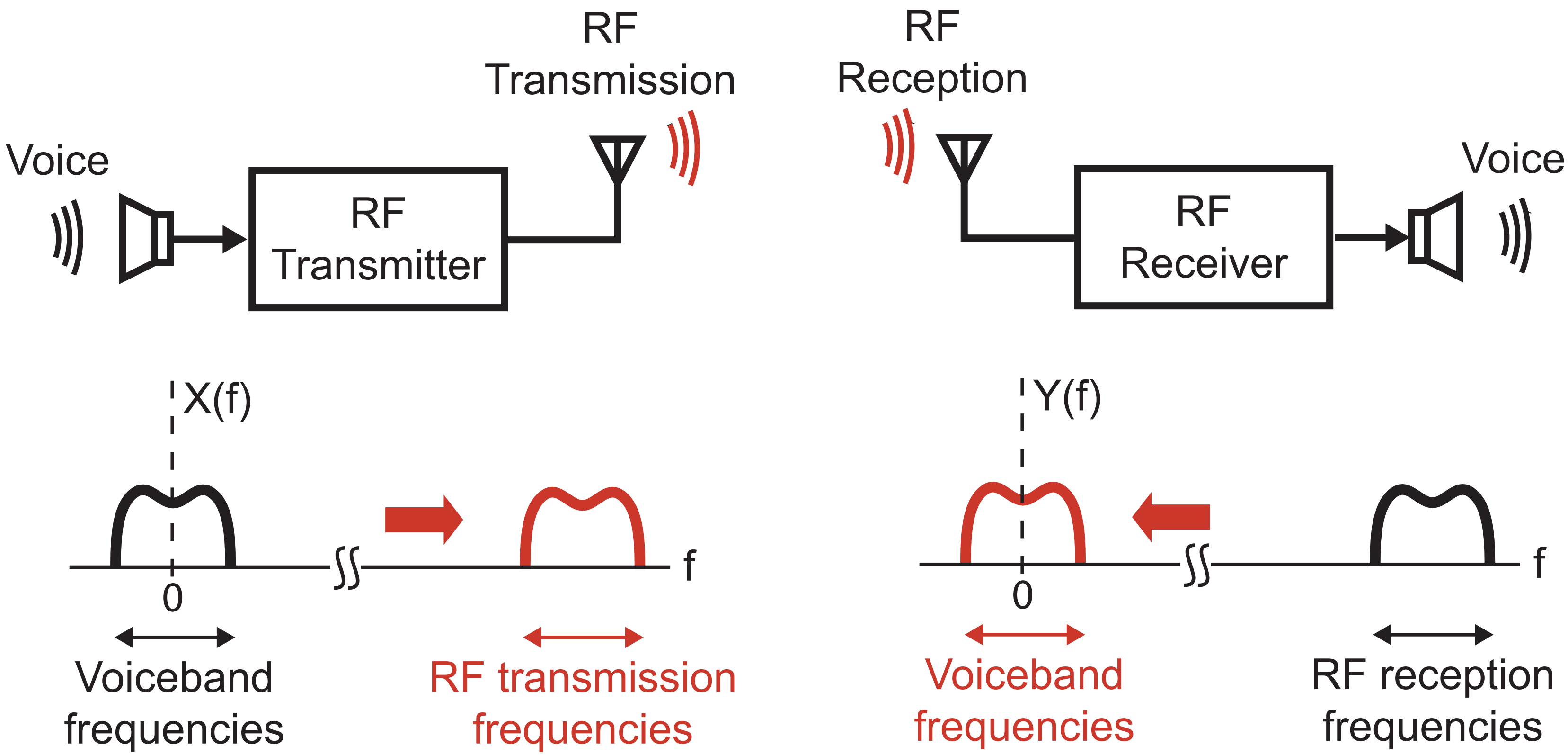


$$X(f) = \frac{A \sin(2\pi f T)}{\pi f}$$

This is called a *sinc* function

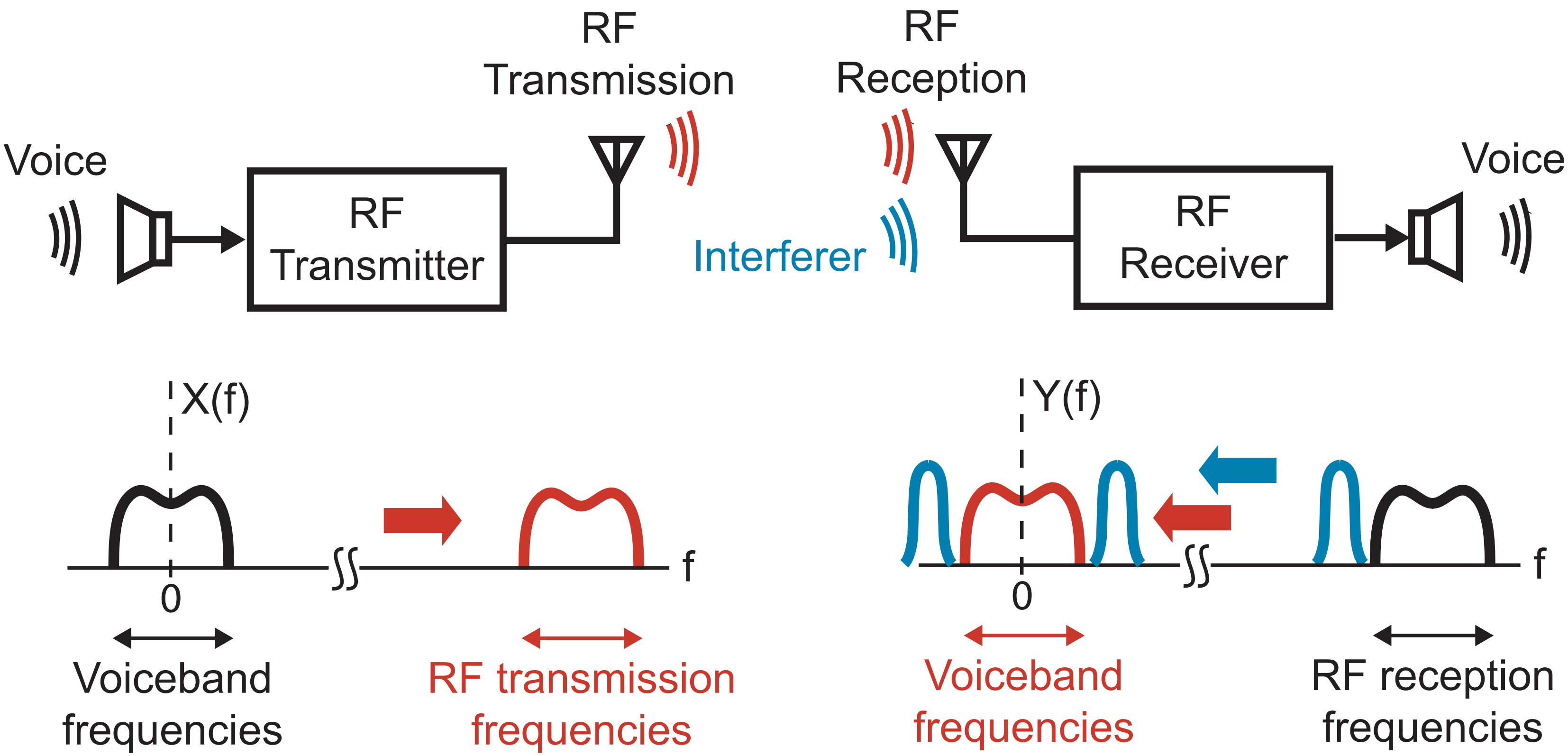


Modulation



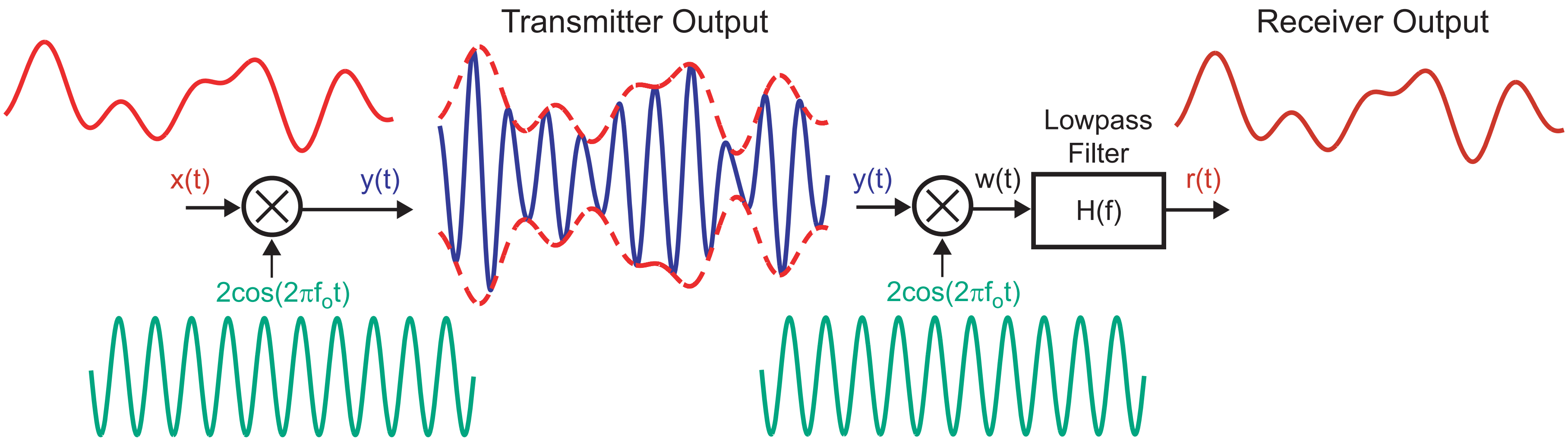
- **Modulation is used to change the frequency band of a signal**
  - **Enables RF communication in different frequency bands**
    - **Used in cell phones, AM/FM radio, WLAN, cable TV, ….**

Filtering



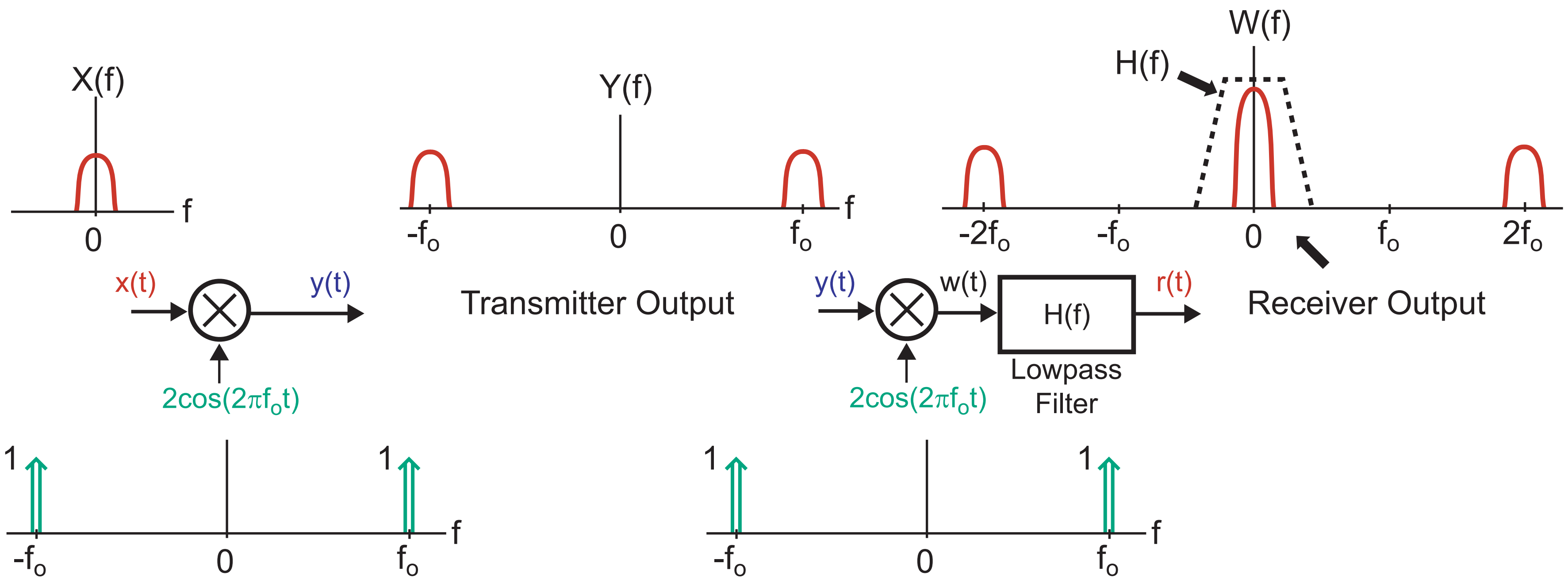
- **Filtering is used to remove undesired signals outside of the frequency band of interest**
  - **Enables selection of a specific radio, TV, WLAN, cell phone, cable TV *channel* …**
  - **Undesired channels are often called interferers**

AM Modulation and Demodulation



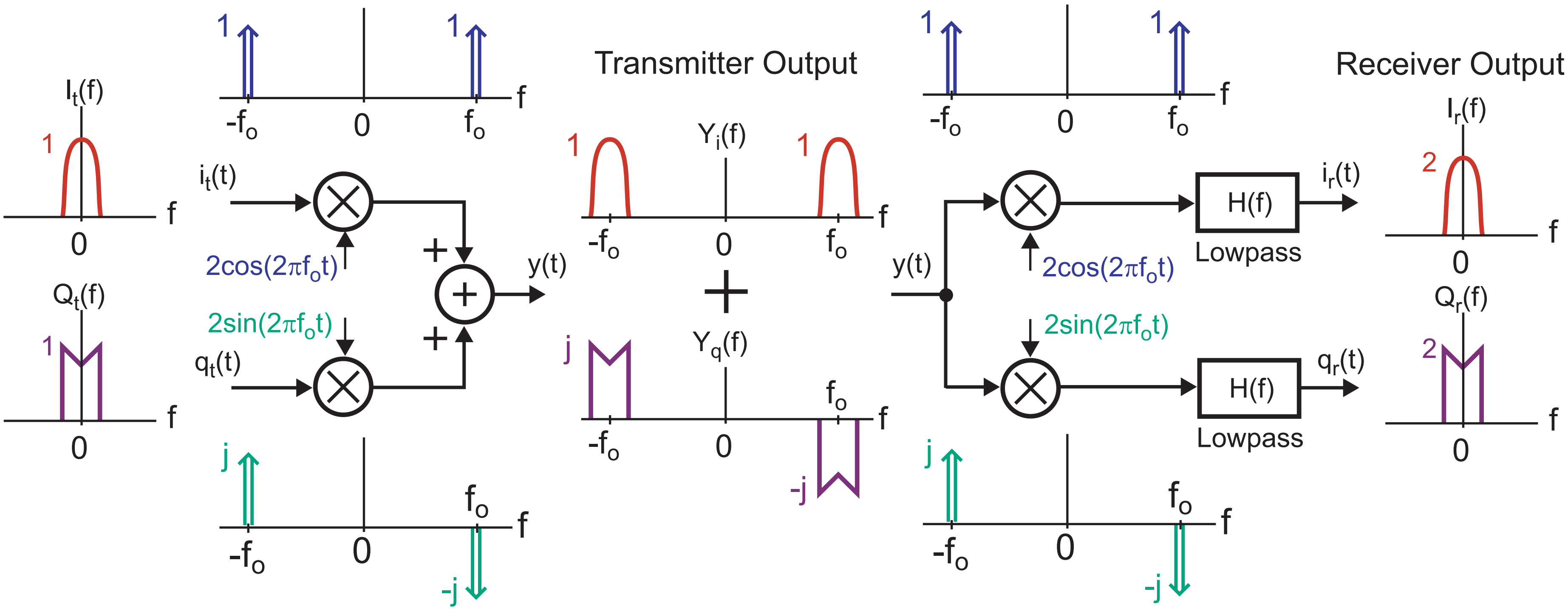
- **Multiplication (i.e., *mixing*) operation shifts in frequency**
  - Also creates undesired high frequency components at receiver
- **Lowpass filtering passes only the desired *baseband* signal at receiver**

Frequency Domain Analysis



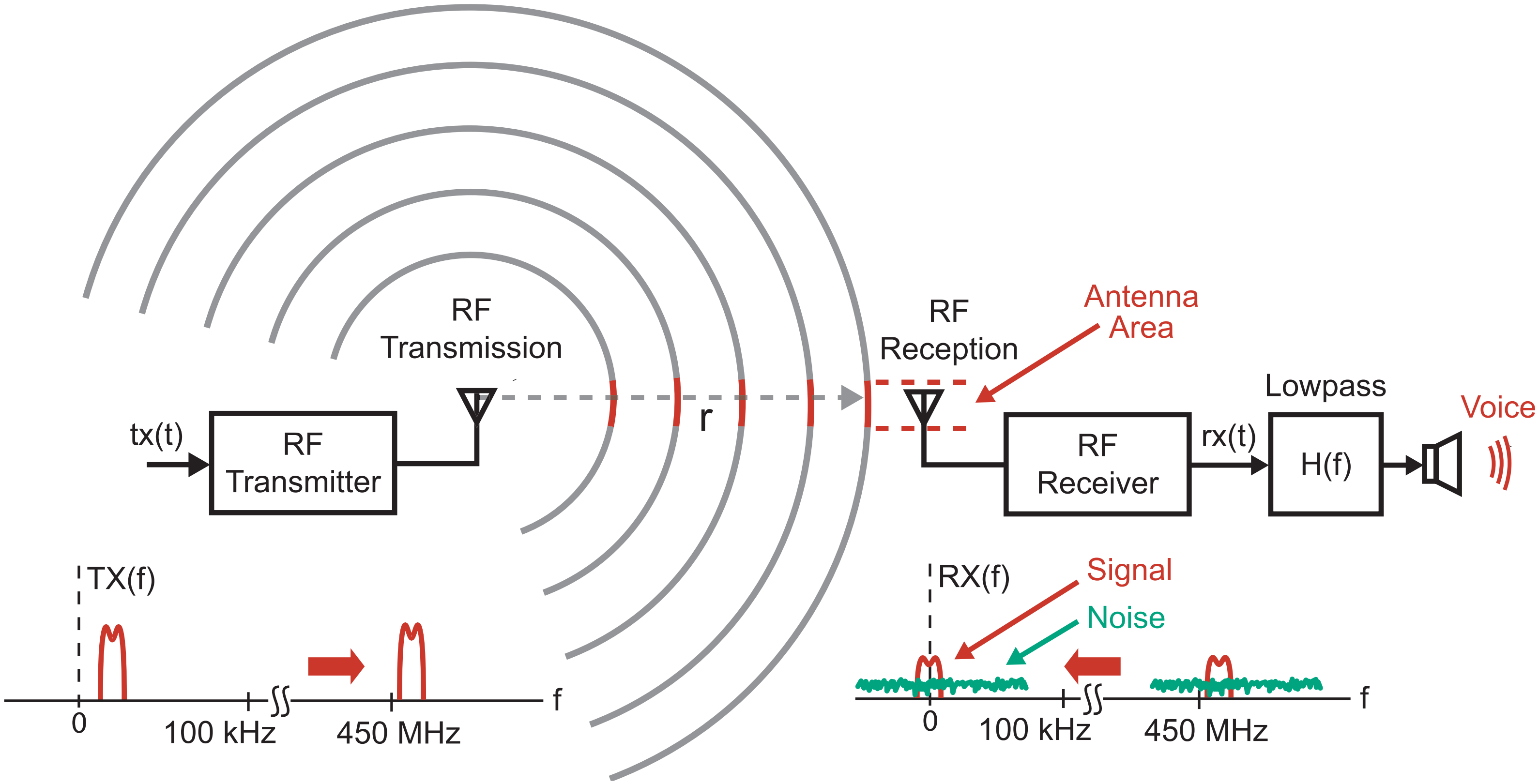
- **When transmitter and receiver local oscillators are matched in phase:**
  - Demodulated signal *constructively* adds at baseband

I/Q Modulation



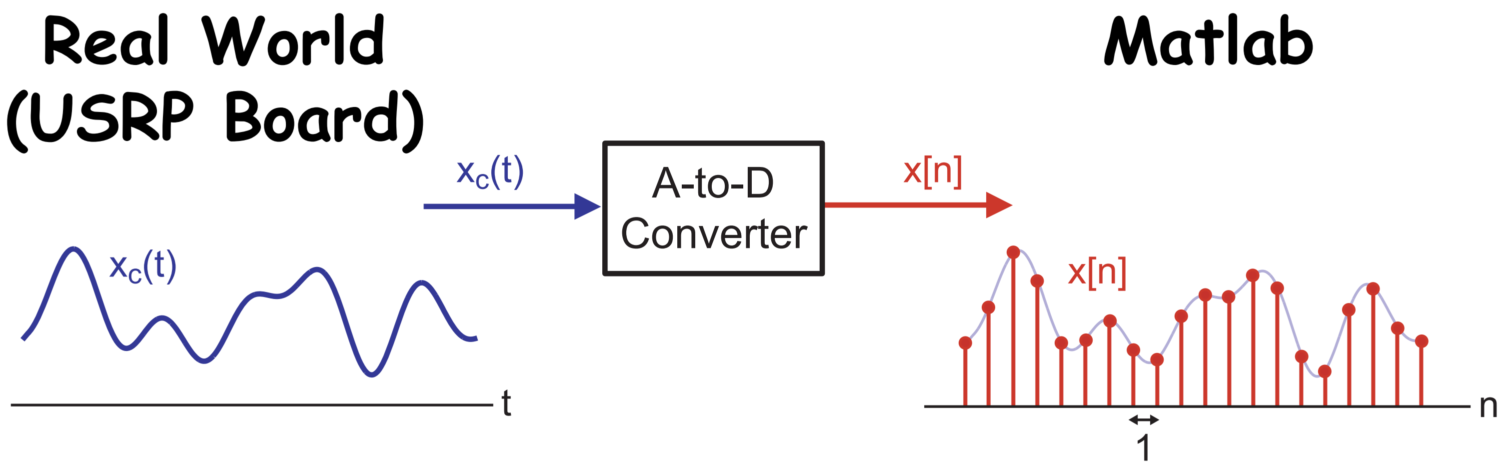
- Modulate with *both* a cosine and sine wave
  - I and Q channels can be broadcast over the *same* frequency band
- I/Q modulation allows twice the amount of *information* to be sent compared to basic AM modulation with same *bandwidth*

Energy Transfer in Wireless Communication



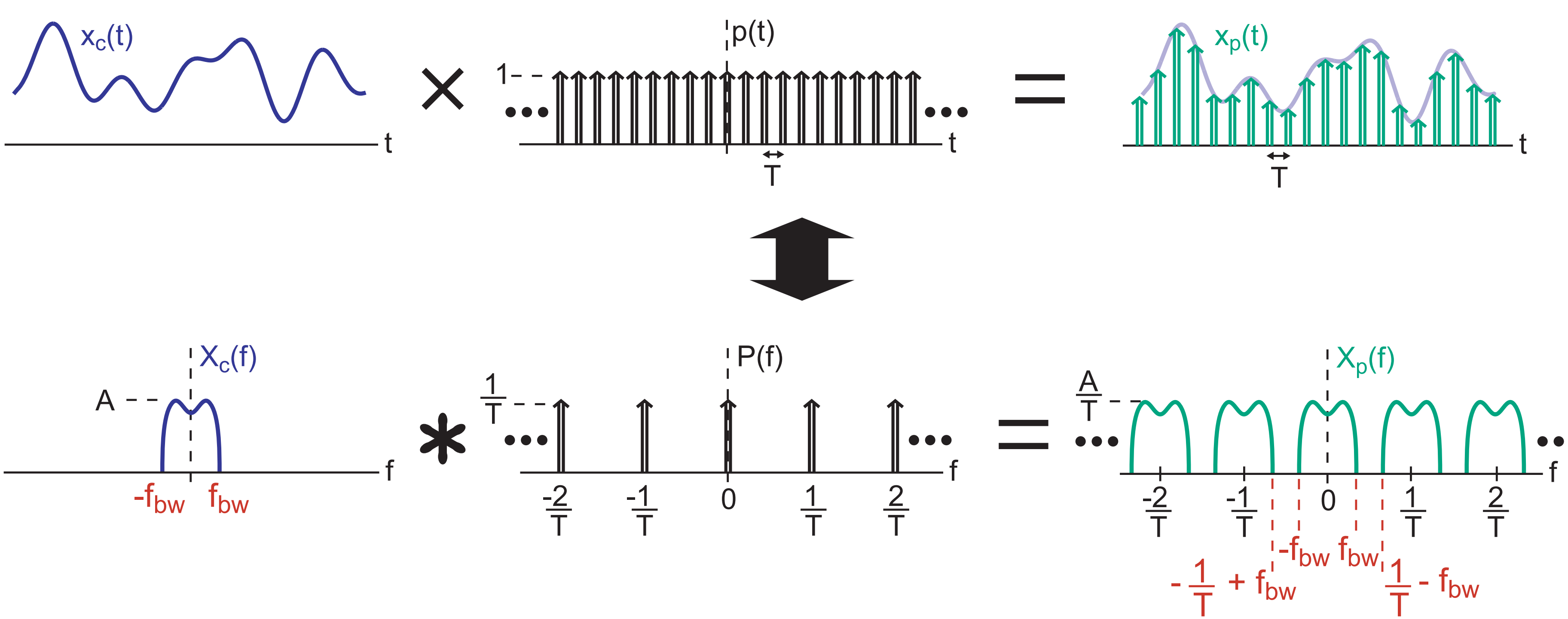
- Receiver antenna is limited in its ability to capture transmitter energy according to its *area* and *distance, r,* from transmitter
- Noise in the receiver causes corruption
  - Amount of corruption depends on signal-to-noise ratio

The Need for Sampling



- The boundary between *analog* and *digital*
  - Real world is filled with *continuous-time signals*
  - Computers (i.e. Matlab) operate on *sequences*
- Crossing the analog-to-digital boundary requires *sampling* of the continuous-time signals

The Sampling Theorem



- Overlap in frequency domain (i.e., aliasing) is avoided if:

$$\frac{1}{T} - f_{bw} \geq f_{bw} \quad \Rightarrow \quad \boxed{\frac{1}{T} \geq 2f_{bw}}$$

- We refer to the minimum *1/T* that avoids aliasing as the *Nyquist* sampling frequency

The Discrete-Time Fourier Transform

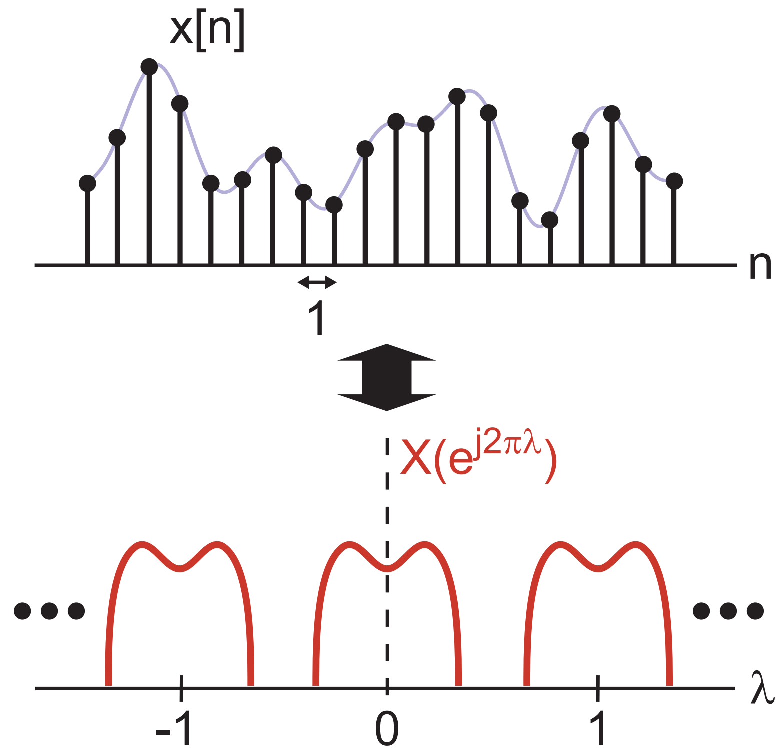
- Allows us to deal with *non-periodic, discrete-time* signals
- Frequency domain signal is *periodic* in this case

$$x[n] \Leftrightarrow X(e^{j2\pi\lambda})$$

- Where:

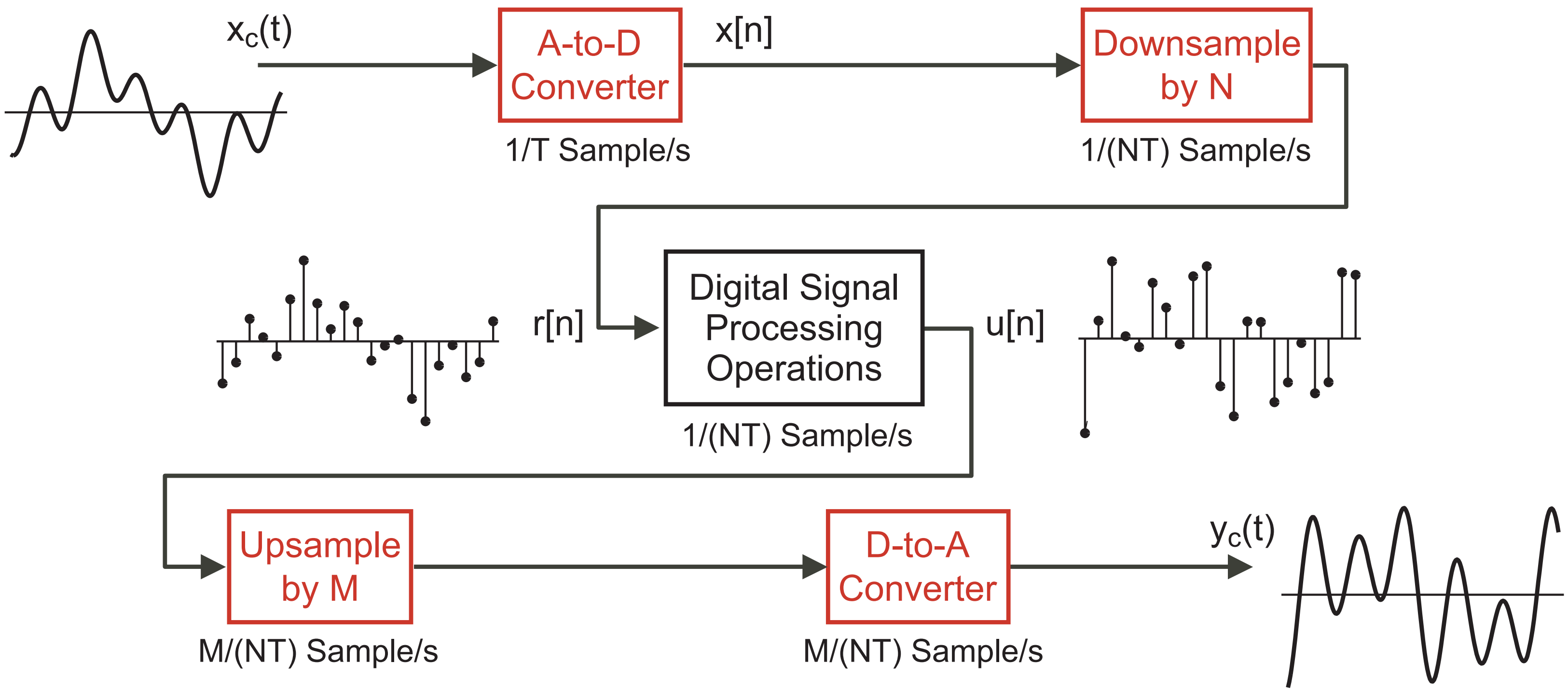
$$x[n] = \int_{-1/2}^{1/2} X(e^{j2\pi\lambda}) e^{j2\pi\lambda n} d\lambda$$

$$X(e^{j2\pi\lambda}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j2\pi\lambda n}$$



Note: *fft* function in Matlab used to compute *DTFT*

Digital Processing of Analog Signals

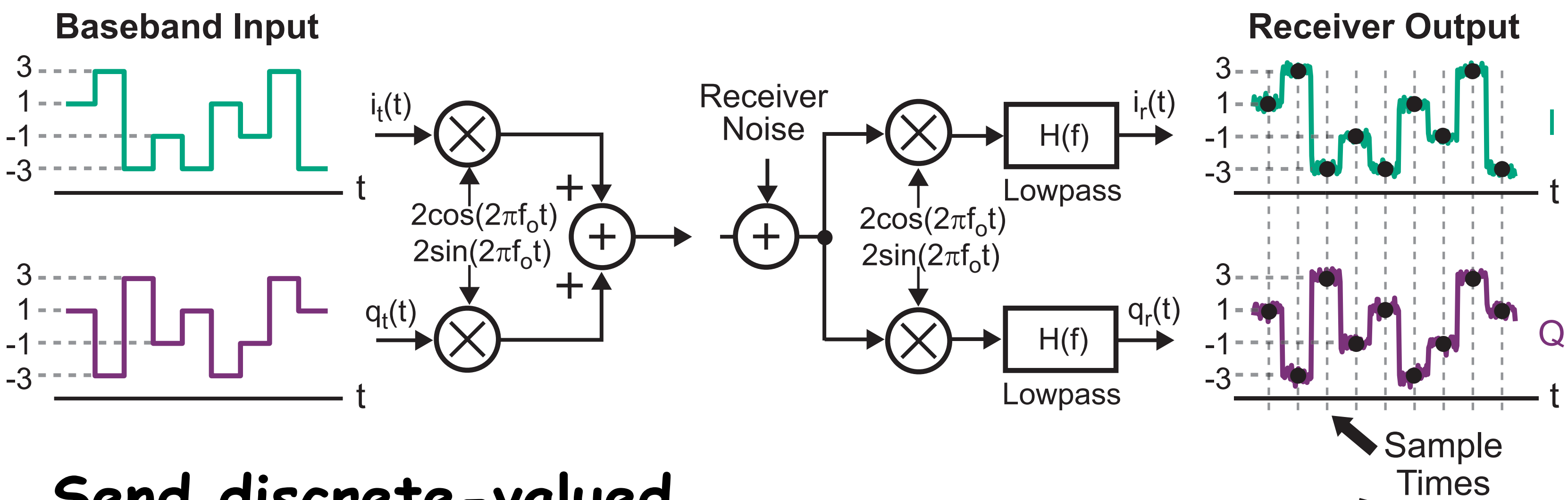


- *Digital* circuits can perform very complex processing of *analog* signals, but require
  - Conversion of analog signals to the digital domain
  - Conversion of digital signals to the analog domain
  - Downsampling and upsampling to match sample rates of A-to-D, digital processor, and D-to-A

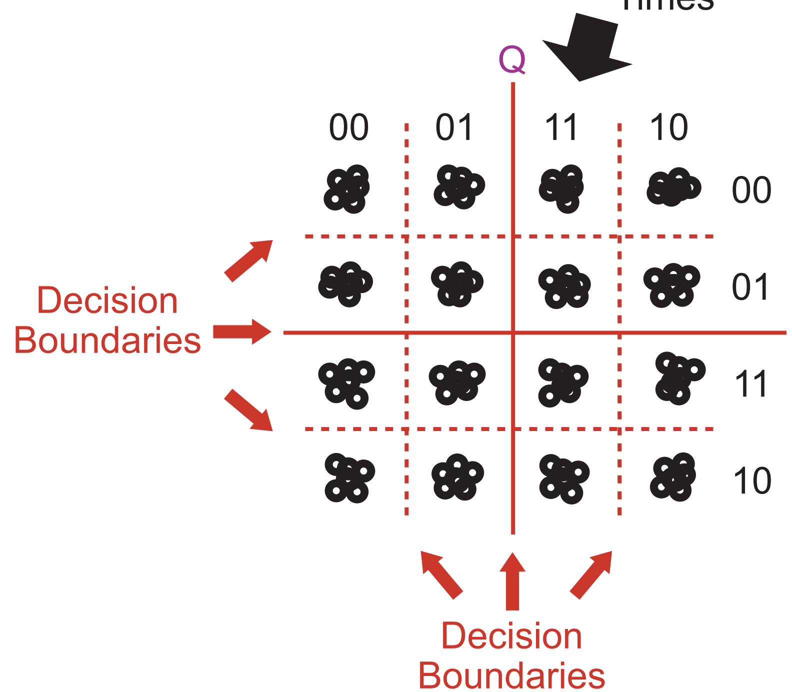
Advantages of Digital Processing

- Digital components correct small analog errors at each processing step
  - We can build large, reliable systems despite non-ideal components and the presence of bounded noise
- We can accommodate more precision by representing information with longer sequences of symbols
  - Except for the conversion steps, we can use simple digital components do achieve arbitrary precision in processing
- We abstract out the notion of "real time" when converting to sequences of discrete values
  - The speed of intervening digital processing steps is independent of the speed of conversion steps (e.g., we can combine many analog streams into a single high-speed digital stream).

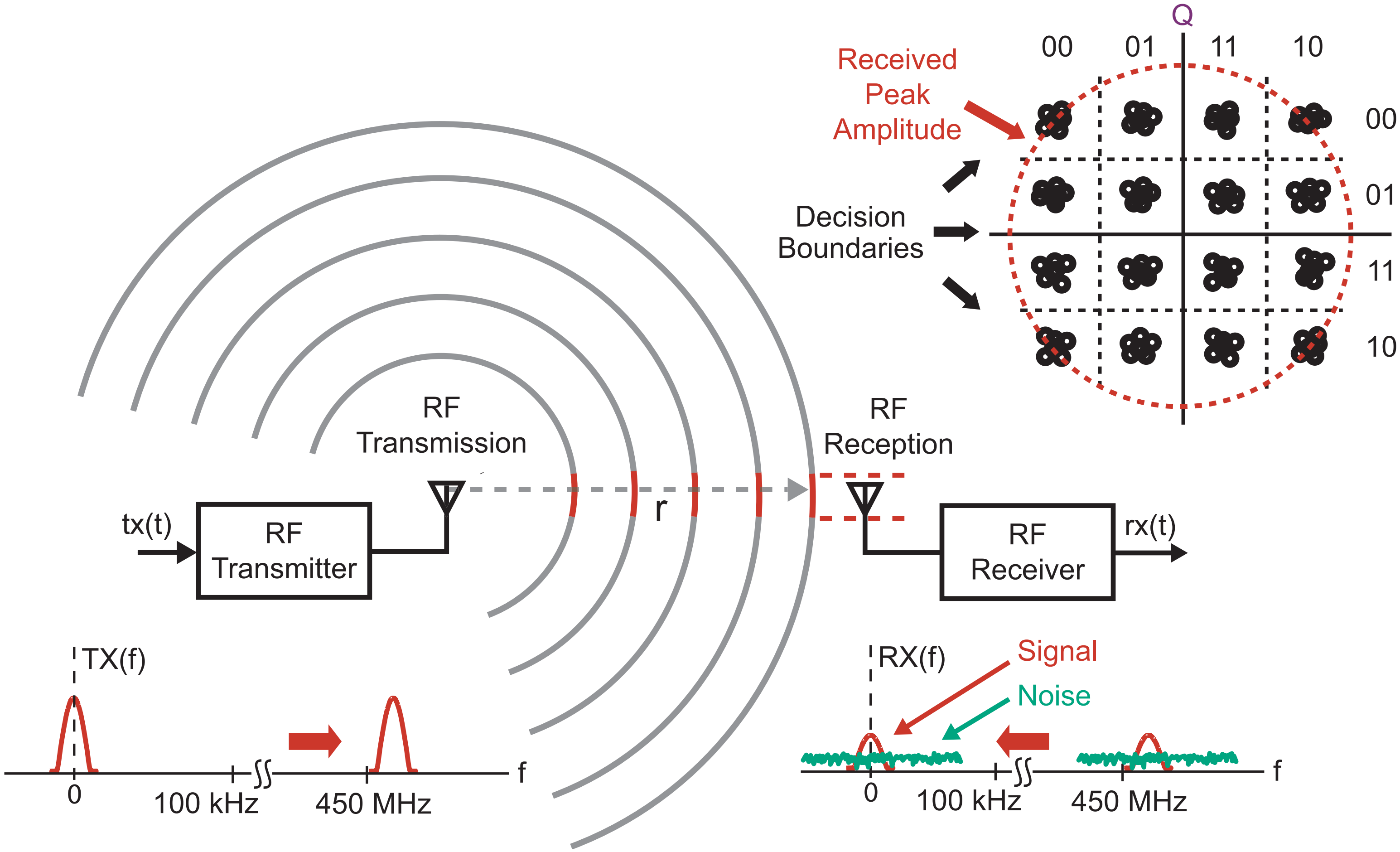
Digital Modulation



- Send discrete-valued symbols across an analog communication channel
- Match I/Q samples to their corresponding symbols based on decision regions
  - Provides noise margin

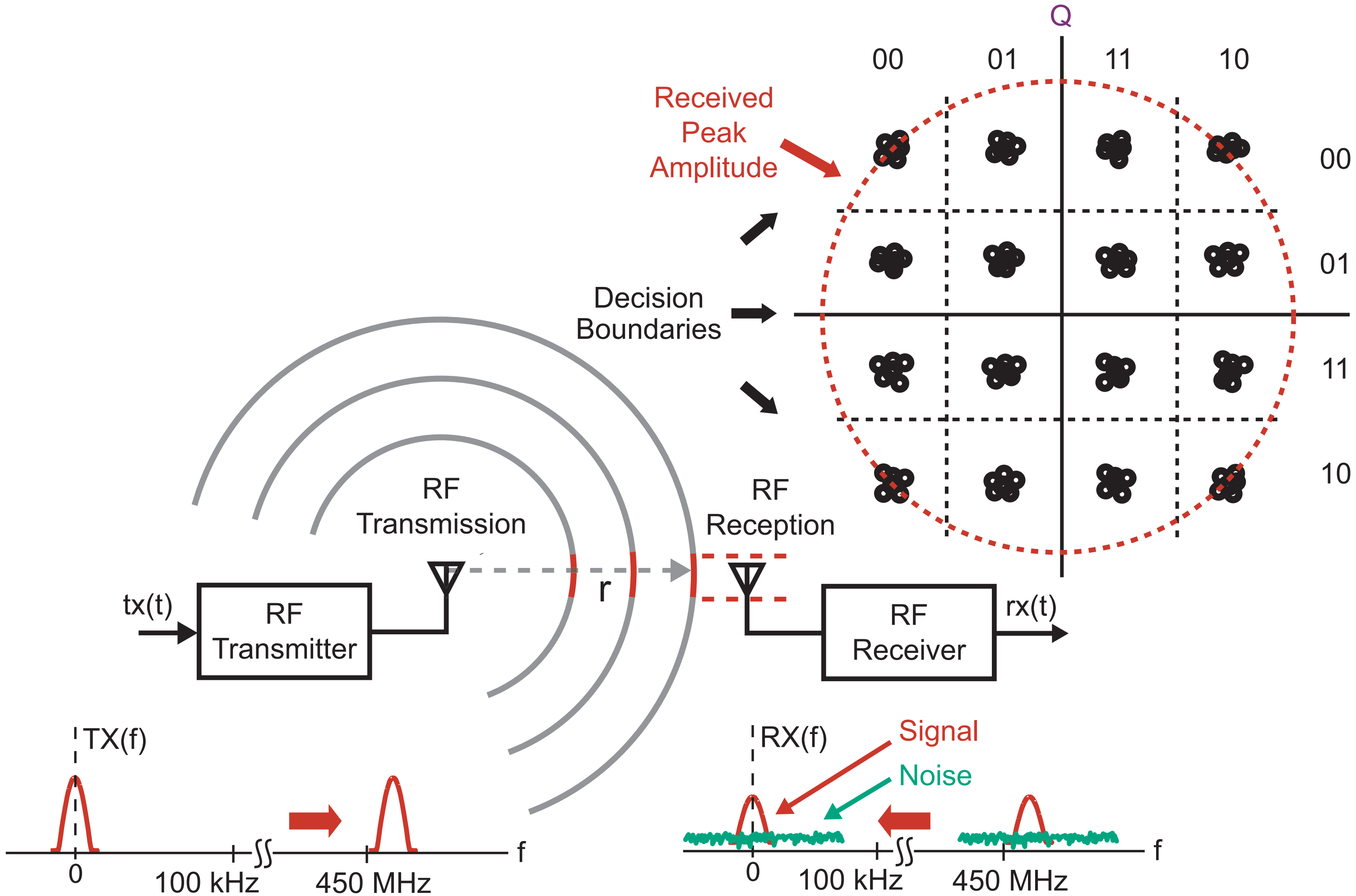


Impact of SNR on Receiver Constellation



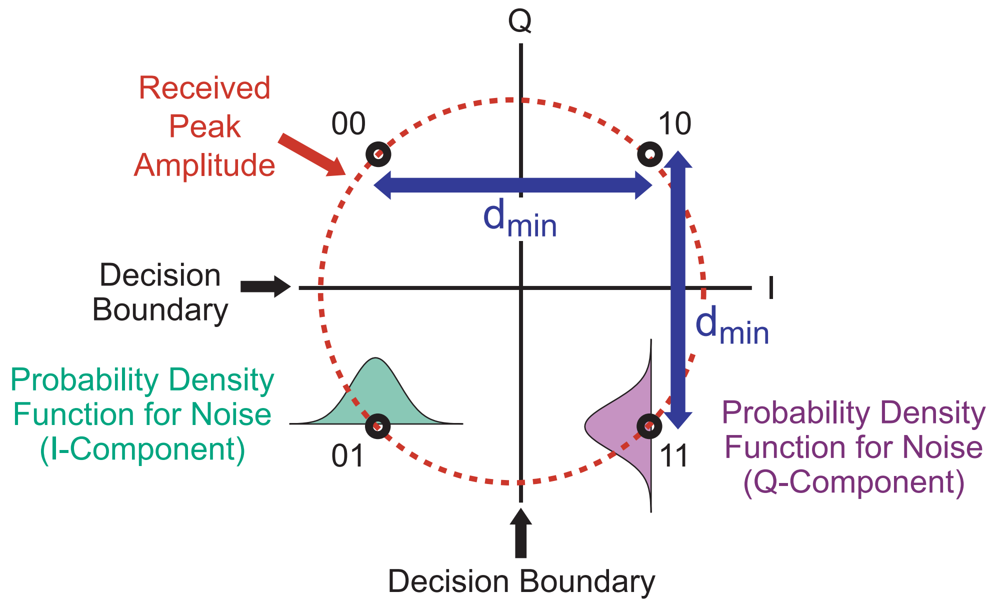
- SNR influenced by transmitted power, distance between transmitter and receiver, and noise

Impact of Increased Signal on Constellation



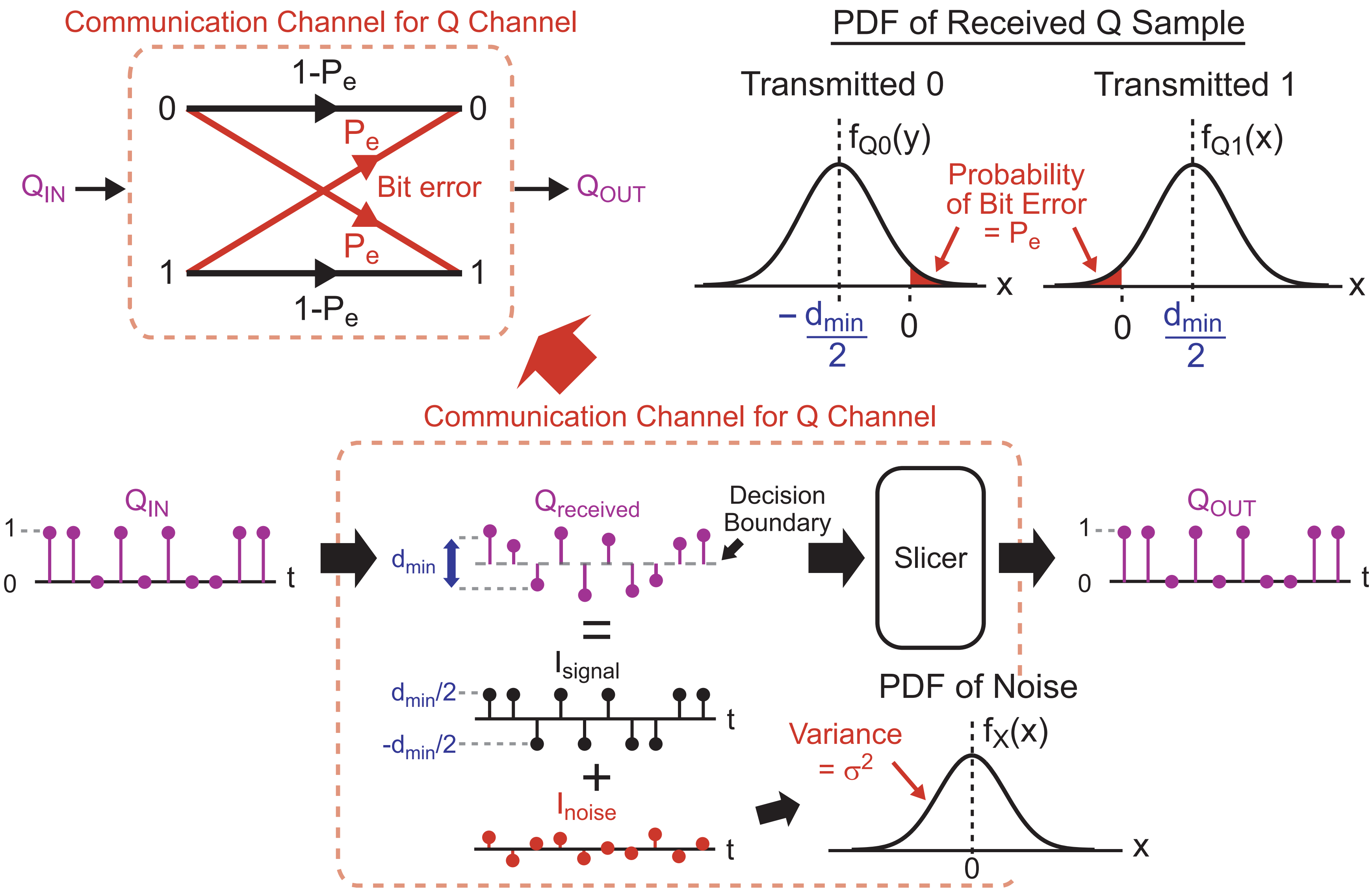
- Increase in received signal power leads to increased separation between symbols
  - SNR is improved if noise level unchanged

Quantifying the Impact of Noise



- Minimum separation between symbols: d_{min}
- PDF of noise: zero mean Gaussian PDF
  - Variance of noise sets the spread of the PDF
- Bit errors: occur when noise moves a symbol by a distance more than dmin/2

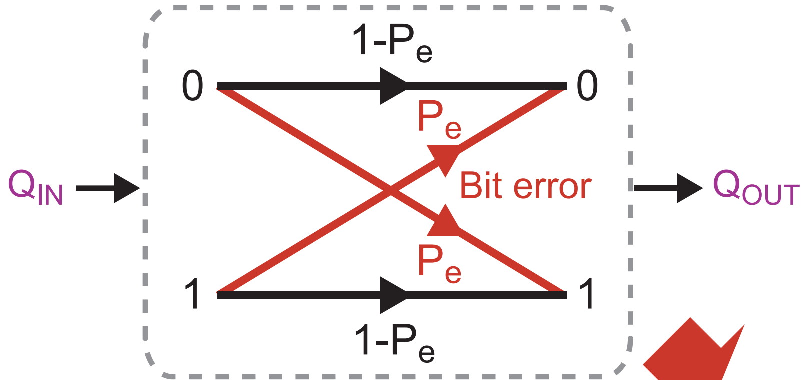
The Binary Symmetric Channel Model



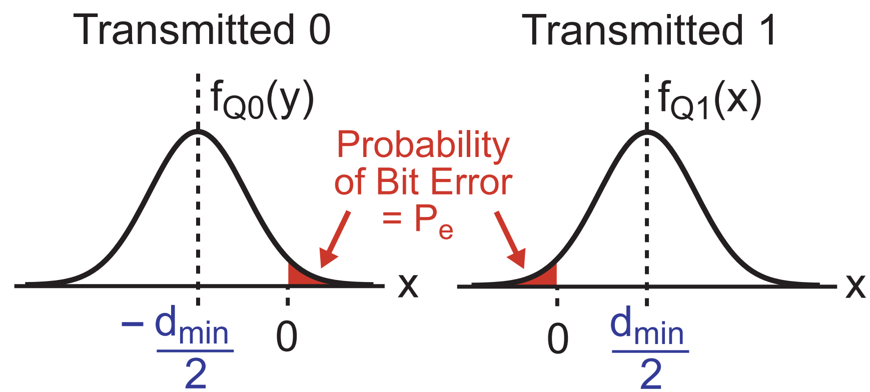
- **Provides a binary signaling model of channel**

Resulting Bit Error Rate Versus SNR

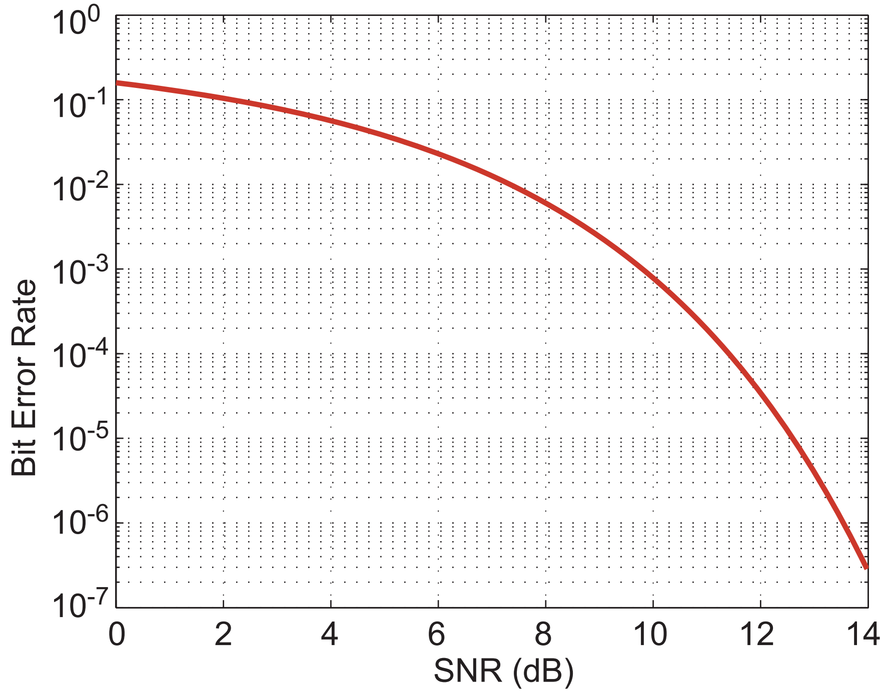
Communication Channel for Q Channel



PDF of Received Q Sample



Bit Error Rate versus SNR for Q Channel



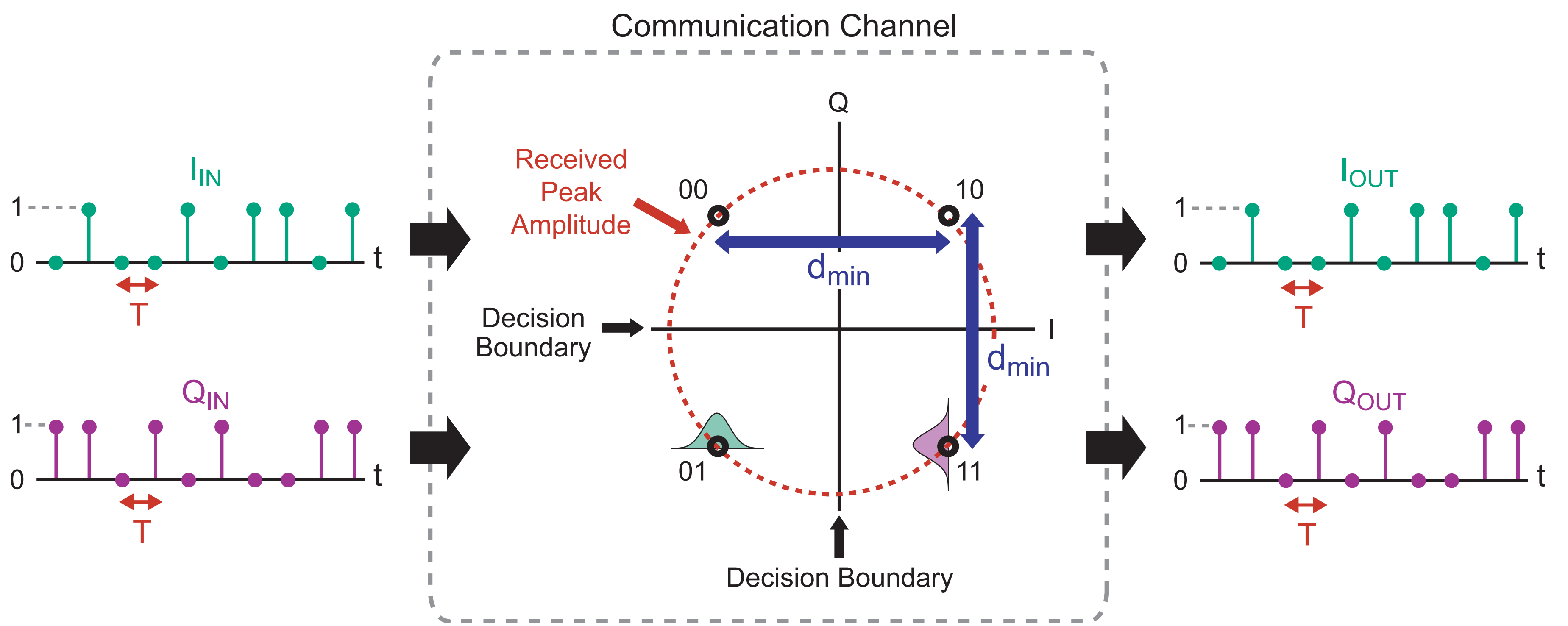
Note:

- **Bit Error Rate = P_e**
- **SNR (dB) =**

$$10 \log \left(\frac{(d_{min}/2)^2}{\sigma^2} \right)$$

- **Gaussian PDF for noise**

Shannon Capacity



- In 1948, Claude Shannon proved that
  - Digital communication can achieve arbitrary low bit-error-rates if appropriate *coding* methods are employed
  - The capacity of a *Gaussian channel* with bandwidth *BW* to support arbitrary low bit-error-rate communication is:

$$C = BW \log_2(1 + SNR) \text{ bits/second}$$

Summary

- The Fourier Transform provides a powerful tool for analysis of sampling, modulation, and filtering
- The digital abstraction provides a practical implementation framework for complicated systems
  - Analog signaling is highly susceptible to noise
  - Digital signaling provides noise margin
- We can represent a digital communication channel with a binary signaling model
  - Bit errors are quantified in terms of the signal-to-noise ratio of the overall channel
- Claude Shannon introduced the concept of using coding methods to achieve arbitrarily low bit error rates across practical communication channels